

Liquidity in the forward exchange market

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Abstract

The forward foreign exchange market is modelled within the framework of a limited participation two-country model and then simulated using the artificial economy methodology. The new model improves on the standard two-country cash-in-advance model in a number of ways. It gets closer to the observed lack of autocorrelation in spot returns and it helps to explain the persistence in the forward discount. However, it cannot account for the relative volatilities of spot returns and the forward discount. Finally, the model goes some distance in explaining the forward discount bias puzzle but falls short of resolving it. © 2001 Elsevier Science B.V. All rights reserved.

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1. Introduction

In this paper, we present an improvement on the standard two-country cash-in-advance (CIA) model. Our focus is to explain the behaviour of the forward exchange rate. However, the framework is a general equilibrium one so it has some useful insights into other variables, particularly the spot exchange rate. Specifically, we construct a limited participation household model along the lines

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of Lucas (1990) and Grilli and Roubini (1992). An explicit forward market that throws light on the importance of liquidity effects in this market is added. We also allow for endowment as well as monetary shocks.

One of the striking, but little, noted features of the standard CIA model is that it predicts that spot returns are autocorrelated if the underlying international money growth differential is itself autocorrelated. Indeed, if the international money growth differential is a long memory process, the CIA model even predicts that this property transfers onto spot returns. This is clearly unsatisfactory because spot returns are close to white noise while the money growth differential is certainly not. Our model succeeds in seriously weakening this implausible relationship.

A number of writers, including Macklem (1991), Backus et al. (1993) and Bekaert (1994, 1996) have tried to explain the “puzzle” that forward exchange rate premia are persistent and may even be fractionally integrated (e.g., Baillie and Bollerslev, 1994; Masih and Masih, 1998). One of the contributions of the paper is that we succeed in making clear how this can arise from within the standard CIA model, if the international money growth differential is persistent.

It is often argued (e.g., Flood and Rose, 1998) that standard CIA models simply cannot explain the extent of volatility of spot returns. This is undoubtedly true and we will reinforce this point. However, the model we have constructed is capable of mimicking observed volatilities under certain circumstances.

According to the standard CIA model, the forward discount is an unbiased predictor of realised future spot returns. It is well known that this is not the case (Engel, 1996; Sibert, 1996; Bekaert, 1996). Our model brings the theory closer to the data but it still falls far short. One advantage of our analysis is that it clarifies what is required of a satisfactory theory. The structure of the paper is as follows. Section 2 provides a critical background to the standard CIA theory. In Section 3, our model is introduced. Section 4 reports the result of simulating an artificial economy. Section 5 gives directions for future research.

2. Background

2.1. General formulation

The assumptions of the standard two-country CIA model are well known and Hodrick (1987) provides an excellent summary. There are two features of the standard model that are important for this paper. The first is that though goods are paid for in cash, the model is quite silent on the means of payment for the purchase of assets. The second is that assets are priced and traded in each time period *after* real and monetary shocks are made known. The standard model can be crystallised in the following four ‘efficiency’ conditions.

Firstly, the spot exchange rate is provided by a purchasing power parity condition. This is derived from the notion that the bilateral real exchange rate equals the marginal rate of substitution between home and foreign goods.

$$S_t = \frac{U_{2t}/P_t^2}{U_{1t}/P_t^1} \quad (1)$$

where S_t is the spot exchange rate, measured as the home price of foreign currency; U_{1t} and U_{2t} are the period t marginal utilities of the home and the foreign goods, respectively; P_t^1 and P_t^2 are the period t nominal prices of home and foreign goods, respectively.

The second and third efficiency conditions are the familiar home and foreign nominal bond-pricing formulae. These are, of course, not peculiar to just cash-in-advance models but are shared by many monetary models.

$$q_t^i = \beta E_t \left[\frac{U_{it+1}/P_{t+1}^i}{U_{it}/P_t^i} \right] \quad i = 1, 2 \quad (2)$$

where q_t^i are the home and foreign nominal prices of one-period bonds, β is the subjective rate of discount and E_t is the expectations operator conditional on time t .

The final efficiency condition is the no-arbitrage identity of covered interest parity that is a feature of all models.

$$\frac{F_t}{S_t} = \frac{q_t^2}{q_t^1} \quad (3)$$

where F_t is the one period ahead forward foreign exchange rate expressed as the home price of foreign currency.

The four efficiency conditions provide us with home and foreign nominal bond prices along with spot and forward exchange rates. In this discussion, we are only concerned with the behaviour of the exchange rates. The importance of the two bond prices lies in the fact that they enable us to derive the forward exchange rate through covered interest parity.

2.2. Some revealing approximations

The theory outlined in Section 2.1 is too general to identify the predicted stochastic properties of spot and forward exchange rates. To progress further, we need to make concrete assumptions about the functional form for utility as well as the sources of uncertainty. Assume that utility is intertemporally separable with an iso-elastic equal shares instantaneous utility function. Next, assume that home and foreign consumption growth follow mean-stationary stochastic processes with normally distributed i.i.d. innovations, which have the same variance for both countries. The assumption that the innovation variance is the same in both

countries affects nothing of substance and simply eases exposition. Assume that the money growth processes are defined analogously. In this case, it is helpful to develop some explicit notation. Let u_{t+1}^i , $i = 1, 2$ be a normally distributed i.i.d. innovation with the same variance σ_π^2 for both countries and π_t^i , $i = 1, 2$ be the conditional expectation of country i 's money growth at time t .

The following assumption is significant. Like a number of previous writers, most notably Engel (1992, 1996), we point out that the observed covariances of real and nominal shock innovations are typically zero. This is an empirical regularity in the asset pricing literature generally but we have replicated this yet again on a G7 data set, which we report in Section 4 of the paper. The combination of this stylised fact and iso-elastic *time inseparable* utility makes the standard CIA model a very weak basis for explaining time-varying risk premia. We also assume that cross-country real and nominal covariances are also zero. This involves very little loss of generality but evidence is provided for it in Section 4 anyway.

We are now in a position to examine the predicted stochastic properties of the spot and forward exchange rates in the standard CIA model. Using the cash-in-advance-for-goods constraint, Eq. (1) enables us to write the spot return as:

$$\log\left(\frac{S_{t+1}}{S_t}\right) = (\pi_t^1 - \pi_t^2) + u_{t+1}^1 - u_{t+1}^2 \quad (4)$$

Hence, spot rate returns are equal to the money growth differential. The *only* way in which the standard CIA model will successfully predict that spot returns are non-autocorrelated, as they typically are, is if the underlying money growth differential is white noise. Indeed, the standard model predicts that the stochastic properties of the money growth differential (whatever they are) are mapped directly onto spot returns. There is abundant evidence that the money shocks are, in fact, persistent. Moreover, we explore the suggestion in Section 5 that money shocks may even follow long memory processes. If the money growth differential also has a long memory, it is alarming that the standard CIA model predicts that this property would also be held by spot returns.

The properties of the forward rate are obtained indirectly through bond prices. Using Eqs. (2) and (3) and the properties of the lognormal distribution, we obtain the following expression for the forward discount¹

$$\log\frac{F_t}{S_t} = \pi_t^1 - \pi_t^2 \quad (5)$$

¹The standard CIA model with homoscedastic forcing processes can generate a *constant* risk premium as well as a constant non-convexity term. They are both zero because of our assumptions on the variance-covariance matrix of the innovations to exogenous shocks.

which is the difference between conditionally expected home and foreign money growth. Hence, the forward discount will have a persistent autocorrelation function if the money growth differentials are persistent.

A comparison of Eqs. (4) and (5) reminds us of the traditional vehicle (e.g., Moore, 1994) for testing for unbiasedness, namely regressing spot returns, $\log(S_{t+1}/S_t)$, on the forward discount. The standard CIA theory clearly implies that the forward discount should be an unbiased predictor of spot returns.

The standard CIA theory also implies that speculative profits and the forward discount should be orthogonal, *irrespective of the nature of the forcing processes for money*. Backus et al. (1993) suggest that a useful summary statistic for the extent of the forward market bias is provided by the estimated variance of the fitted values from the regression of $\log(S_{t+1}/F_t)$, on the forward discount.

3. The model

To address the challenges posed in Section 2, we develop a limited participation model in a two-country world along the lines of Lucas (1990), Fuerst (1992), Grilli and Roubini (1992) and Christiano et al. (1997). Limited participation models differ from the standard CIA two-country models as follows. Asset portfolios cannot be adjusted costlessly. This idea is implemented by specifying that all portfolio decisions are made before the realisation of money shocks, both foreign and domestic. This is significant because assets, as well as goods, must be purchased with cash that must be accumulated in advance.

The specific contribution made in this paper is to extend this class of models to allow for forward foreign exchange contracts. The sluggish portfolio adjustment behaviour, which we have modelled for all other assets including spot exchange rates, does not affect forward contracts. Since there are no margin requirements, the model drives an additional liquidity ‘wedge’ between spot returns and the forward discount. The formal specification of the model is derived in the Appendix. We again crystallise its main features into four ‘efficiency’ conditions. These should be compared directly with the analogous conditions for the standard CIA model (Eqs. (1)–(3)).

Firstly, purchasing power parity no longer holds in any conventional form. Instead of Eq. (1), the spot exchange rate is determined as follows:

$$S_t = \frac{E_t(U_{2t+1}/P_{t+1}^2) q_t^1}{E_t(U_{1t+1}/P_{t+1}^1) q_t^2} \quad (6)$$

In contrast to Eq. (1), the price-weighted marginal utilities are expected values. This reflects the fact that all decisions are made before shocks are known. The appearance of the bond price ratio in Eq. (6) is its most important feature. It

follows, firstly, from the fact that goods arbitrage can only be mediated through money. However, unlike the standard CIA model, money has another opportunity cost because of its use in purchasing bonds. Goods arbitrage diverts monetary resources away from asset markets and this effect appears as the bond price ratio in Eq. (6). This is useful in helping to explain the lack of autocorrelation in spot returns because it breaks down the linear link with money shocks, which is revealed in Eq. (4). In addition, real shocks will not cancel out as they did for the standard CIA case. The second reason why Eq. (6) improves on the standard CIA model in explaining spot returns is that this model predicts that their volatility will be higher because of the presence of asset prices on the right hand side.

The second and third efficiency conditions are bond-pricing conditions and are analogous to Eq. (2).

$$E_{t-1} \left[\beta \frac{U_{it+1}}{P_{t+1}^i q_t^i} - \frac{U_{it}}{P_t^i} \right] = 0 \quad i = 1, 2 \quad (7)$$

Eq. (7) embodies the sluggish portfolio assumption of the model. It would be identical to Eq. (2) if expectations were taken at time t instead of $t - 1$. Because portfolios are set before shocks are known, the Fisher equation, even allowing for a risk premium, only holds on average. Unlike in the standard CIA model, the bond prices remain implicit. The additional source of non-linearity can be interpreted as giving rise to a ‘liquidity’ premium. This adds further volatility to bond prices and through Eq. (6) to the spot exchange rate. The final efficiency condition need not be repeated here because it is simply the covered interest parity condition of Eq. (3).

It would be pleasing if the approximations, which we applied to the standard CIA model of Section 2.2, could be extended here. However, the non-linearity of Eq. (6) and (7) makes this almost completely unrewarding. What we can expect is that the simple link between money shocks and spot exchange rates is eroded, both because of the presence of real shocks and because of the non-linearity. In addition, a new liquidity wedge is driven between the spot and forward markets. To clarify the model any further, we need to conduct numerical simulations.

4. Empirical and model evidence

4.1. Calibration

We calibrate the model discussed in Section 3 and compare the moments generated from the model with those in quarterly data. There are nine parameters to choose. The discount rate, β , is assumed to be $(1.03)^{-0.25}$, which is based on an

annual real rate of interest of 3%; a value commonly used in the literature. The share of consumption of home produced goods in the domestic agent's utility function, α , is set equal to 0.5, a value estimated in Stockman and Tesar (1995). These parameters remain constant in the various experiments we simulate. It emerges that risk aversion has major effects in the liquidity-constrained model. The coefficient of relative risk aversion is allowed to vary from 2 to 10.

We need to specify parameters of the exogenous consumption and money growth rate processes.² We hypothesise that at the very most a second-order four-variable vector autoregression should capture the basic features of these growth processes. We estimate six vector autoregressions: each VAR consist of four variables. They are endowment and money growth from the US and each of the remaining G7 countries. We compare the Schwarz Information Criterion for the general VAR(2) model and various restricted versions of that process. We were unable to reject the hypothesis that endowment and money growth are both described by scalar AR(1) processes for the country pairs considered. In addition, the correlations between innovations in consumption and money growth within and between countries are jointly statistically insignificant from zero. In the light of this, we decided to use the same parameters for the forcing processes for both the home and foreign countries. This implies, for example, that the money growth differential is also an AR(1) process with the same parameter as the individual home and foreign money AR(1) processes.

Changing the parameters of the consumption growth processes *do not affect* the statistics of interest. For all experiments we assume the following values for the parameters of both the home and foreign AR(1) consumption growth processes; the unconditional mean is set equal to 0.6%, the standard error of the AR(1) process is assumed to be 0.8% and first-order autocorrelation coefficient is set equal to 0.21. These numbers are representative of G7 countries over the period 1976–1993.

In all our experiments, we set the unconditional mean of both home and foreign money growth equal to 1.4%/quarter: it does not affect any of our results. We also set the standard error of the AR(1) money growth processes to be equal to 0.97%/quarter.³ These parameter values are representative of G7 countries over the period 1976–1993. In contrast, the first-order autocorrelation coefficient in the AR(1) process for the money growth processes has major effects on the summary

² All our empirical results are available upon request. We use quarterly G7 data from 1976–1993 to replicate stylised facts about exchange rates and to calibrate parameters in the exogenous shock processes. All series are available from Datastream. We assume that the consumption series is seasonally adjusted real consumption from the OECD Main Economic Indicators and the money series is seasonally adjusted M2 from national central banks. We use exchange rates for the US against other G7 countries.

³ The assumption about σ_π is innocent. Varying this parameter simply varies all standard deviations of the summary statistics proportionately. Nothing else is affected.

statistics. This is because the money growth differential process shares this coefficient. We allow the AR(1) coefficient in money growth to vary from 0.1 to 0.9 in both countries. We base our results on the mean of 1000 replications of the linear–quadratic solution to both standard and liquidity-constrained models.

4.2. Results

Risk aversion has major effects on the persistence of spot returns in the liquidity-constrained model. To illustrate this we set the AR(1) coefficient in home and foreign money growth to be 0.78, which is representative of the G7 countries, while the coefficient of relative risk aversion is allowed to vary from 2 to 10. A summary of the effects of changing relative risk aversion on the first-order autocorrelation coefficient of spot returns is shown in Fig. 1, where the solid line represents the liquidity-constrained model and the dashed line represents the standard model.

Risk aversion does not affect the first-order autocorrelation coefficient of spot returns in the standard model. It is constant at 0.72: this is close to the autocorrelation coefficient of the money growth differential and Eq. (4) makes it clear why this is so. Obviously, this does not correspond to the almost zero level of the autocorrelation of spot returns found in the data. The liquidity-constrained model is very encouraging in addressing the problem of the overprediction of the

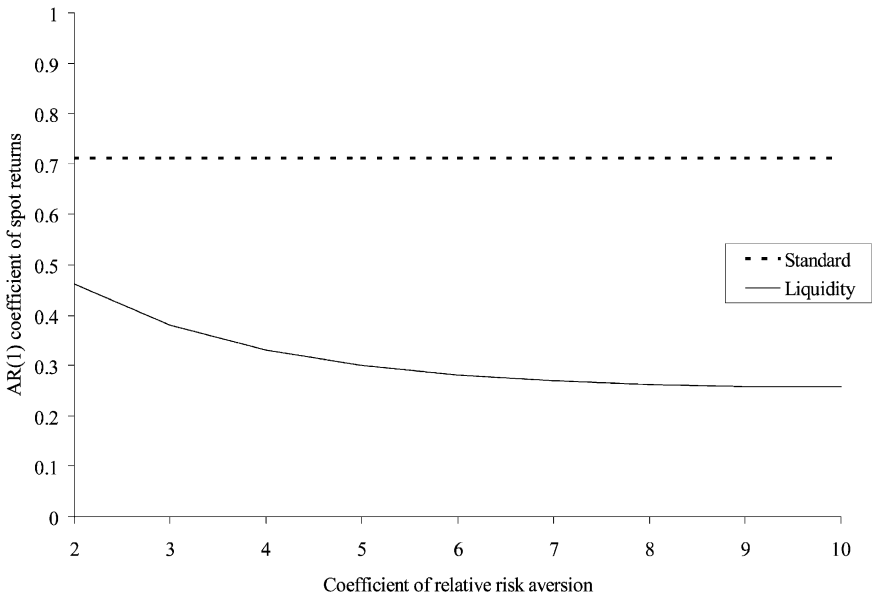


Fig. 1. Persistence of spot returns.

persistence of spot returns in the standard model. At high levels of risk aversion this model produces a first-order autocorrelation coefficient of spot returns of 0.26. This is closer to the zero value, which is empirically observed, than the high value that is predicted by the standard CIA model.

The persistence of the forward discount has eluded many previous studies with the notable exceptions of Bekaert (1994, 1996). To investigate this, we set the coefficient of relative risk aversion at 10 and vary the AR(1) coefficient of the money growth processes from 0.1 to 0.9. The summarised results are graphed in Fig. 2. For *both* the standard and liquidity-constrained models, the AR(1) coefficient for the forward discount reflects that of the money growth processes. In order to match the persistence of the forward discount that is typically found in the data, the AR(1) coefficient in the money growth processes needs to be set in the 0.7–0.8 range.

A key indicator of the success of a model in explaining the forward market is its ability to account for the high empirical standard deviation of the fitted values from the regression of $\log(S_{t+1}/F_t)$, on the forward discount. Following Backus et al. (1993), we refer to this fitted value as the ‘expected profit from currency speculation’. To illustrate this we set the AR(1) coefficient in the money growth

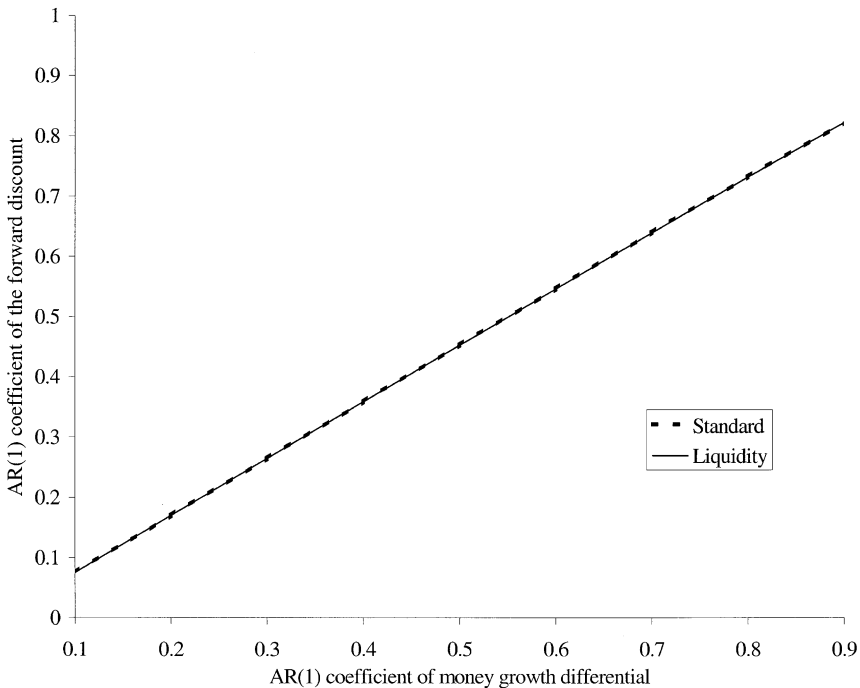


Fig. 2. Persistence of the forward discount.

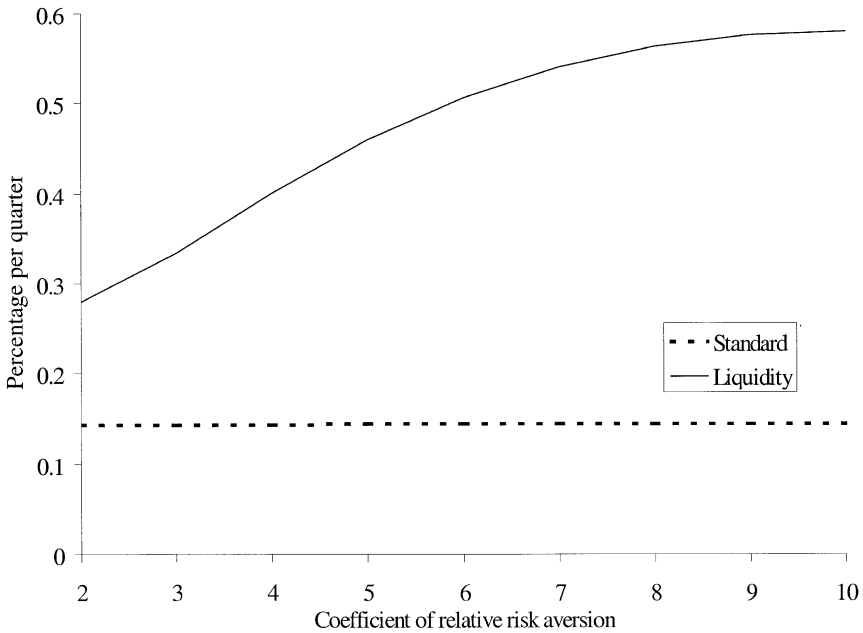


Fig. 3. Standard deviation of the expected profit from currency speculation.

processes to be 0.78 and the coefficient of relative risk aversion is allowed to vary from 2 to 10. The summarised results are reported in Fig. 3.

As risk aversion rises, this standard deviation rises to 0.54%/quarter in the liquidity-constrained model: this is a fivefold improvement on the standard model. However, this value is still very far from the 2–3%/quarter that is usually found in the data (Hodrick, 1987; Backus et al., 1993). A striking feature is that risk aversion does not affect the standard deviation of the expected profit from currency speculation in the standard model. This is a general result with regard to volatilities in the standard model. For example, in the standard model risk aversion does not affect the standard deviations of spot returns and the forward discount. In the liquidity-constrained model, on the other hand, as agents become more risk averse the standard deviation of spot returns rises to values of 6.9%/quarter. This is a value typically found in the data. However, the standard deviation of the forward discount also rises with risk aversion in the liquidity model.

5. Conclusion

So long as risk aversion and the persistence of the money growth differential are both high, the new model improves on the standard model in explaining the

lack of persistence in spot returns while continuing to explain the persistence in the forward discount. It fails miserably to explain the high volatility of spot returns in relation to the volatility of the forward premium. It goes some distance in explaining the forward discount ‘bias’ but much more work needs to be done on this aspect. The asset pricing literature is currently emphasising the importance of time-inseparable preferences. On its own, this is unlikely to help (Bekaert, 1996) but it would be well worthwhile exploring in combination with the limited participation framework of this paper.

A useful feature of our results is that we argue that the way to explain the persistence of the forward discount is through the persistence of the money growth differential. This needs to be explored further. For example, it is worth examining the impact of fractionally integrated processes for money growth in these models. Baillie and Bollerslev (1994) argue that the forward discount is not just persistent but has a long memory. They estimate the order of (fractional) integration for monthly data from 1974 to 1991 for Canada, Britain and Germany (with the dollar as numeraire). Estimated values lie in the range 0.45–0.77. Eq. (5) immediately suggests a possible reason for this: that money growth differentials are themselves fractionally integrated. There is surprisingly little direct work on this but the available studies are strongly suggestive. The standard CIA model proposes a simple quantity theory relationship between prices and money with unit velocity. It is reasonable, therefore to examine ARFIMA studies of goods price inflation in order to obtain clues about the underlying properties of money shocks. Baillie et al. (1996) model monthly CPI inflation from 1948 to 1990 for G7 and three high-inflation countries—Argentina, Brazil and Israel. They estimate ARFIMA–GARCH models for each country. The significant findings are that the estimated order of integration of inflation is significantly greater than 0 for all but one of the countries (the exception is Japan). For the remaining G7 countries, the estimate lies between 0 and 0.5 while for the three high-inflation countries, it is approximately 0.59. In a separate study, Hassler and Wolters (1995) examine monthly CPI inflation for the US, Germany, Britain, France and Italy over the 1969–1992 period. Again, they find clear evidence of long memory processes. The estimated orders of integration vary from 0.4 to 0.57, which are higher than those found by Baillie et al. This is almost certainly accounted for by the fact that the analysis by Baillie et al. is more richly specified. The most direct evidence, on the long memory properties of money shocks, is provided by Porter-Hudak (1990). She models monthly data for United States M1, M2 and M3 during the period 1947–1986. Her focus is to assess whether the money stock series are fractionally integrated at seasonal frequencies. Her framework prevents her from being able to identify separate orders of integration for each seasonal frequency. She estimates an overall order of integration, which varies (depending on monetary aggregate and sample) from 0.402 to 0.721. These estimates apply, of course, to the zero frequency, which is our main interest, as well as to the other seasonal frequencies. These issues need to be effectively addressed in future work.

Appendix: A limited participation model

The households in both countries have the same intertemporal utility function

$$U = \sum_{t=0}^{\infty} \beta^t \frac{\left((c_{it}^1)^\alpha (c_{it}^2)^{(1-\alpha)} \right)^{1-\gamma} - 1}{1-\gamma}, \quad i = 1,2 \tag{A1}$$

where β is a subjective discount factor, α is the share of total consumption that is attributed to domestic goods, γ is the coefficient of relative risk aversion and c_{it}^j is the consumption by country i of country j 's goods.

The agent in the goods market faces the following cash-in-advance constraint

$$N_{it}^j \geq P_t^j c_{it}^j, \quad i = 1,2, \quad j = 1,2 \tag{A2}$$

where N_{it}^j is the amount of money of country j held by the household of country i for transactions in the goods market at time t and P_t^j is the price of country j goods in terms of country j money. The agent faces the following cash-in-advance constraint

$$Z_{it}^1 + S_t Z_{it}^2 \geq q_t^1 B_{it}^1 + S_t q_t^2 B_{it}^2, \quad i = 1,2 \tag{A3}$$

where Z_{it}^j is the amount of money of country j held by the household of country i for transactions in the asset market at time t , S_t is the domestic price of foreign currency at time t , q_t^j is the price of country i 's discount bonds⁴ and B_{it}^j is the total amount of bonds of country j held by the household of country i at time t .

If interest rates are positive, both cash-in-advance constraints will hold with equality. Thus, at the beginning of period $t + 1$, the domestic households holding of domestic currency

$$M_{1t+1}^1 \geq P_t^1 c_t^1 + B_{1t}^1 - F_t G_t^1 \tag{A4}$$

is made up of proceeds from the sale of the endowment, the redemption of the discount bonds and liabilities from forward contracts in the previous period. $G_t^1 > 0$ constitutes the number of “long” forward contracts. The domestic household's holding of foreign currency is

$$M_{1t+1}^2 \geq B_{1t}^2 + G_t^1 \tag{A5}$$

Analogously, the foreign households holding of foreign currency is

$$M_{2t+1}^2 \geq P_t^2 c_t^2 + B_{2t}^2 - G_t^2 \tag{A6}$$

and of domestic currency is

$$M_{2t+1}^1 \geq B_{2t}^1 + F_t G_t^2 \tag{A7}$$

where $G_t^2 > 0$ constitutes a “short” position in forward foreign exchange for the foreign country.

⁴ Since these are discount bonds, the domestic nominal interest rate is defined implicitly through $q = 1/(1 + r)$. The foreign nominal interest rate is defined analogously.

The only role for the government is to have a central bank that engages in open market operations. In each period, the central bank of each country changes the money stock by issuing one-period discount bonds. The bonds are redeemed at the beginning of the next period. Exogenous money growth is given by

$$\frac{M_{t+1}^j}{M_t^j} = (1 + \pi_t^j), \quad j = 1, 2 \tag{A8}$$

where

$$M_{it}^j = N_{it}^j + Z_{it}^j, \quad i = 1, 2, \quad j = 1, 2 \tag{A9}$$

$$M_t^j = M_{1t}^j + M_{2t}^j, \quad i = 1, 2$$

M_{it}^j is the total amount of money of country j held by the household of country i at time t . Exogenous endowment growth is given by

$$\frac{c_{t+1}^j}{c_t^j} = (1 + \mu_{t+1}^j), \quad j = 1, 2 \tag{A10}$$

Equilibrium in the goods market is given by

$$c_t^j = c_{1t}^j + c_{2t}^j \quad j = 1, 2 \tag{A11}$$

Equilibrium in the asset market is given by

$$Z_t^j = Z_{1t}^j + Z_{2t}^j = q_t^j (B_{1t}^j + B_{2t}^j) = q_t^j B_t^j, \quad j = 1, 2 \tag{A12}$$

Equilibrium in the forward foreign exchange market is given by

$$G_t^1 = G_t^2 \tag{A13}$$

We can now formulate the domestic household’s problem. Let $V(\bullet)$ represent the value function. Assuming that the cash-in-advance constraints are binding, the domestic household solves

$$V(M_{1t}^1, M_{1t}^2) = \text{Max} \quad E_{t-1} \left(\text{Max} \left[U \left(\frac{N_{1t}^1}{P_t^1}, \frac{N_{1t}^2}{P_t^2} \right) + \beta E_t V(M_{1t+1}^1, M_{1t+1}^2) \right] \right)$$

$$\text{s.t.} \quad (M_{1t}^1 - N_{1t}^1) + S_t (M_{1t}^2 - N_{1t}^2) \geq q_t^1 B_{1t}^1 + S_t q_t^2 B_{1t}^2 \tag{A14}$$

where expectations are taken over the set of four exogenous stochastic state variables $\{c_t^j, M_{it}^j$ for $i = 1, 2, j = 1, 2\}$. The first maximisation is with respect to N_{1t}^1 and N_{1t}^2 . The second maximisation is with respect to B_{1t}^1, B_{1t}^2 and G_t^1 . The first-order conditions are summarised by the efficiency conditions given by Eqs. (3), (6) and (7).

There is no closed form solution for this non-linear stochastic rational expectations model. Thus, we find an approximate solution using the linear–quadratic

methods of Christiano (1991). This yields optimal linearised rules for the four unknowns, q_t^1 , q_t^2 , S_t and F_t . A technical appendix containing this solution method is available from the authors upon request.

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