Global Financial Management

Valuation of Cash Flows
Investment Decisions and Capital Budgeting

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6.0 Overview:

This class provides an overview of capital budgeting - determining which investments a firm should undertake. The net present value (NPV) rule, which is widely used in practice, is developed and illustrated with several examples. A number of alternative evaluation techniques including internal rate of return and payback period are also illustrated, highlighting potential problems with their use. The NPV technique is illustrated in the context of choosing between mutually exclusive projects and projects with different lives.

6.1 Objectives:

After completing this class, you should be able to:

- Compute the net present value of an investment proposal.

- Explain why the NPV rule leads to optimal decisions

- Compute the internal rate of return of an investment proposal.

- Explain the limitations of the IRR as an investment appraisal criterion.

- Compute the payback period of an investment proposal.

- Determine whether a particular investment proposal should be undertaken.

- Determine which (if any) of a set of investment proposals should be undertaken when the firm is capital constrained.

- Determine which (if either) of two mutually exclusive investment proposals with different lives should be undertaken.

- Compute the appropriate cash flows to use in the NPV analysis.

6.2 Projects and Cash Flows

Every decision the firm makes is a capital budgeting decision whenever it changes the company's cash flows. Consider launching a new product. This involves a phase where the new product is advertised and distributed. Hence the firm will have cash outflows for paying advertising agencies, distributors, transportation services etc. Then, for a period of time, the firm has cash inflows from the sale of the product in the future. Alternatively, consider the decision to make or buy a certain component the firm needs as an input it currently purchases from another company. Making the input requires payments for labor and materials, but saves payments to the supplier, and all these cash inflows and outflows are affected by that decision. Many other decisions affect the company's cash flows:

- Choice of distribution channel

- Purchases of buildings

- Choice of geographical location

- Purchase of another company or sale of a division

- Leasing or buying a certain piece of equipment

- Reducing dividend payments in order to pay down bank debt

The difficulty with making these decisions is that typically many cash flows are affected, and they usually extend over a long period of time. Investment appraisal criteria help us in analyzing capital budgeting decisions by aggregating the multitude of cash flows into one number.

But **which** cash flows? If we decide to make a component, should the cost of the factory building where it is made be included? What about the salary of the sales manager if a new product is launched? The answer to this question is clear and simple: ***All cash flows have to be included in our analysis whenever they are affected by the decision!*** Hence, if launching a new product implies hiring a new sales manager, then her salary is included. If the sales manager would continue to be employed anyway, then her salary is a cash outflow the company would incur even if the product were not launched, and then her salary is not included. Similarly, the factory building may have been there already without any other use for the firm (then don't include it), or it could have been sold (then include foregone cash inflow from not selling it). Alternatively, it may exist, but using it for making a component may force us to lease another building (then include these lease payments). These cash flows are also called *incremental cash flows*, since they always compare the cash flows for a base scenario (do not launch product, do not make component) with an alternative scenario. The differences of the cash flows in the base and the alternative scenario are the incremental cash flows. We denote these incremental cash flows by X_t, where $X_t > 0$ indicates that the firm's cash inflow increases in time t as a

result of the decision, and $X_t < 0$ indicates the opposite. Hence, from a point of view of capital budgeting procedures, a decision is completely characterized by the stream of incremental cash flows:

$$X_0 \quad X_1 \quad X_2 \quad \cdots \quad X_t \quad \cdots \tag{1}$$

Note that $X_t > 0$ does not imply that the firm has a positive cash flow at time t under the alternative scenario, since the base scenario may imply a negative cash flow already. Reconsider the example of make versus buy. In each case will the firm have cash outflows from the purchase, but making may imply lower outflows, so the decision to make rather than outsource would imply positive incremental cash flows in some periods. We will often refer to cash flow streams like (1) as "projects", since the classical problem for capital budgeting was an investment problem. However, *any* decision that is reflected in changes in the company's cash flows can be analyzed using the techniques discussed in this lecture.

Analytically, characterizing the decision by a stream of cash flows as in (1) presents us with two challenges:

- We have to estimate these cash flows X for all periods in the future where the decision under consideration has an impact on the cash flows. This implies forecasting. We turn to this problem in section 6.13 below.

- We have to use some investment appraisal method in order to analyze decisions where X is positive for some periods, and negative for others. We have to understand the *time value of money* in order to proceed correctly. We discuss the solution to this problem in the following sections.

- The incremental cash flows estimated here are typically uncertain, and we have to take into account that some cash flows are certain, whereas others depend on the state of the economy. We return to the problem of risk later in the course. There we shall see that we can take care of the riskiness of projects by using adequate discount rates. In this lecture we take the discount rate r_P appropriate for a project P as given.

6.3 Net Present Value (NPV):

The investment appraisal measure we wish to propose here is the net present value, or NPV. The NPV of a project is defined as the present value of all future cash flows produced by an investment, less the initial cost of the investment.

Let X_t denote the dollar cashflow in time t and N the number of such cashflows. In addition let r_p denote the required rate of return and I the initial investment outlay. The NPV is defined as:

$$NPV = \sum_{t=1}^{N} \frac{X_t}{(1+r_p)} - I \tag{1}$$

In determining whether to accept or reject a particular projected, the NPV decision rule is

Accept a project if its NPV > 0;
Reject a project if its NPV < 0;[1]

In other words, we accept all and only those proposals that have a positive net present value, and reject all others. In order to illustrate the computation of Net Present Values, we consider a series of examples.

Example 1

Consider the following investment proposal:

Year	0	1	2	…	25
	-100	11	11	11	11

Assuming that the required rate of return for this project is $r_p = 10\%$, is this a worthwhile investment?

Applying the NPV rule here requires the calculation of the present value of the future cash flows followed by a comparison with the investment cost of $100 million.

$$NPV = \left[\frac{1 - (1 + r_p)^{-N}}{r_p} \right] \cdot X - 100$$

$$= \frac{1 - 1..1^{-25}}{0.1} \cdot 11 - 100 = \$ - 0.153$$

(2)

Since NPV < 0 we *reject* this proposal.

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[1] You are indifferent if NPV $= 0$. This is a knife edge case and we will not explicitly emphasize this.

6.4 Why Net Present Value?

In this subsection we wish to motivate why accepting all and only positive NPV

proposals is the correct decision rule. Suppose you have the following investment project:

Year	1997	1998	1999	2000
Project Cash Flow	-100.00	-50.00	30.00	200.00

The discount rate is 10%. It is easy to see that the NPV of this project is 29.6:

$$NPV = -100 - \frac{50}{1.1} + \frac{30}{1.1^2} + \frac{200}{1.1^3} = 29.6 > 0 \tag{3}$$

However, what does this number really mean? The 29.6 is exactly the additional amount

of money shareholders can spend today if they take the project. Suppose there is only one

shareholder who owns the above project, and she can borrow and lend at 10%. Then she

can do the following if she takes the project.

- Spend 29.6 today and borrow the money from the bank.

- Repay the loan by using the project cash flows

The point is to see that the project covers her liability from the bank loan completely. To

see this consider the following table:

Year	1997	1998	1999	2000
Project Cash Flow	-100.00	-50.00	30.00	200.00
Loan Cash Flow	129.60	50.00	-30.00	-200.00
Interest	0.00	12.96	19.26	18.18
Balance of account	-129.60	-192.56	-181.82	0.00
Payment to shareholder	29.60	0.00	0.00	0.00

In the first year the shareholder borrows 129.6 from the bank. She uses 100 to cover the investment outlay, and spends the remaining 29.6 on consumption. In the next period she borrows an additional 50 and pays interest at 10%=0.1*129.6=12.96 to the bank. This takes her total debt to the bank to 192.56. Only in 1999 does she start to repay the loan, still accruing interest on her outstanding balance. However, at the end of 2000 she has repaid the loan completely. Hence, had she not taken this project, she would have been worse off, since she could not have spent the 29.6 on consumption.

Now, turn the argument around and suppose the project had a cash flow of 150 in the year 2000, everything else remaining the same. Then the previous table would become:

Year	1997	1998	1999	2000
Project Cash Flow	-100.00	-50.00	30.00	150.00
Loan Cash Flow	92.04	50.00	-30.00	-150.00
Interest	0.00	9.20	15.12	13.64
Balance of account	-92.04	-151.24	-136.36	0.00
Payment to shareholder	-7.96	0.00	0.00	0.00

Now the project has a negative NPV of -7.96. This means that the shareholder has to cut her consumption budget by 7.96 if she wants to take the project, since the project can only repay a bank loan of 92.04 now.

Hence, if shareholders take positive NPV projects, then they can consume more than they could without the project. If they accept negative NPV projects, they have to cut consumption in order to be able to finance the project.

6.5 More than two alternatives

In many cases, a firm will be faced with a choice of between more than two alternatives. For example, a firm may be considering whether to construct an office building or a shopping mall on a parcel of land, or to sell the land, or deciding whether to refurbish an old apartment building or turn it into a parking garage, or leave it in its current condition. In this case, the NPV rule is to undertake the project with the largest NPV, so long as it is positive.

Example 2

A manufacturer is considering purchasing one of two new machines, A and B. The cash flows of each of buying the two new machines are represented below on a time line. These are the incremental cash flows relative to a base scenario where the manufacturer simple keeps the old machine. The required rate of return is 10 percent. Since these decisions are mutually exclusive, which proposal (if any) should the manufacturer choose?

Buy Machine A						
Year	0	1	2	3	4	5
Cash Flow	-3,000	1,000	1,000	1,000	1,000	1,000

Buy Machine B						
Year	0	1	2	3	4	5
Cash Flow	-2,000	700	700	700	700	700

The NPV computations are:

$$NPV_A = \frac{1-1.1^{-5}}{0.1} \cdot 1,000 - 3,000 = \$790.79$$

(4)

$$NPV_B = \frac{1-1.1^{-5}}{0.1} \cdot 700 - 2,000 = \$633.55$$

Since these are mutually exclusive decisions and both have NPV > 0, we take the decision with the highest NPV. Machine A is thus the preferred alternative.

The rationale for this procedure is easy to see. Effectively, we can break down one decision into two decisions here. The first decision is to purchase machine A (the alternative scenario) rather than keeping the old machine (the base scenario). The NPV of this decision is $790.79>0, hence this decision generates a positive NPV and we accept it. The next decision is to purchase machine B rather than machine A. This decision generates incremental cash flows that can easily be computed from the tables above as:

Buy Machine B rather than Machine A						
Year	0	1	2	3	4	5
Cash Flow	1,000	-300	-300	-300	-300	-300

Hence, calculating the NPV of this gives us $-157.24<0, hence this decision is incorrect, since it generates a negative NPV. Note that starting with project B would give us a positive NPV for buying machine B, and also a positive NPV for buying machine A rather than machine B, hence we come to the same conclusion. We can summarize as:

It is optimal to make decisions that generate positive net present values of their incremental cash flows. If there are more than two alternatives, it is optimal to choose the alternative that generates the highest NPV.

6.6 Comparing Projects with Different Lives

Suppose a firm with a required rate of return of 10% is considering the acquisition of a new machine to produce its product. It is deciding between Machine A and Machine B. Machine A has a useful life of the 3 years and machine B has a useful life of 5 years. The present values of cash inflows and outflows over the lifecycle of each machine are as follows:

Machine	PV	Machine life
A	2,000	3 Years
B	3,000	5 Years

 Should the firm choose Machine A or Machine B?

Machine B has a higher PV of cash inflows relative to outflows, but it also has a useful life of 5 years versus the 3 year life of Machine A. Suppose that the output produced by these machines is required for a long period of time. Since machine A is going to be used to produce the firm's output, it is reasonable to assume that it will be replaced at the end of year 3 and thus its NPV above is understated. Assume that the firm will use the machine indefinitely. Hence, at the end of the machine's useful life it will be replaced by another identical machine. Machines of type A are replaced on a 3-year cycle and machines of type B will be replaced on a 5-year cycle.

When we calculate the NPVs of the two alternatives it is important that the decision is to reinvest each machine. NPVs relate to the decision to use a particular machine, not to the physical machine as such. One way to compute the relevant NPV is to compare the annual equivalent cash flows of the two alternative projects. Machine A has a PV of $2,000, but at the required return of 10% we would be indifferent between $2,000 at the beginning of period zero and the *annual equivalent* (AE) of:

$$AE_A = \frac{2000}{\frac{1-1.1^{-3}}{0.1}} = \$804.27 \tag{5}$$

at the end of each of the 3 years. This is because

$$A_n = \left[\frac{1-(1+i)^{-n}}{i} \right] \cdot R \tag{6}$$

so that

$$R = \frac{A_n}{\frac{1-(1+i)^{-n}}{i}} \tag{7}$$

Since we assume that the machine is going to be used indefinitely, this implies that we will receive the annual equivalent cash flow of $AE_A=\$804.25$ indefinitely.

In general, the annual equivalent cash flow is given by:

$$AE = \frac{NPV}{\frac{1-(1+i)^{-n}}{i}} \tag{8}$$

The annual equivalent cash flow of machine B is thus given by:

$$AE_B = \frac{NPV}{\frac{1-(1+i)^{-n}}{i}} = \frac{3000}{\frac{1-1.1^{-5}}{0.1}} = \$791.4 \tag{9}$$

Therefore, we can now compare the annual equivalent cash flows of the two proposals and the decision rule is to accept the proposal with the highest annual equivalent cash flow. Here $AE_A > AE_B$ so the firm should accept project A.

The rule is that for mutually exclusive projects with different lives it is not appropriate to compare the PVs of cash flows of one investment cycle directly. We should, instead, convert these PVs to annual equivalent cash flows (AE) where:

$$AE = \frac{NPV}{\frac{1-(1+i)^{-n}}{i}} \tag{10}$$

and take the project with the highest AE. This applies to cases where the firm is considering one type of machine that is to be replaced indefinitely or an alternative type of machine that is to be replaced indefinitely.

6.7 Alternative Evaluation Techniques

This section outlines several alternatives to the NPV rule. These evaluation techniques include:

- Internal Rate of Return (IRR)
- Payback Period
- Profitability Index

6.8 Internal Rate of Return (IRR)

The internal rate of return, IRR, of a project is the rate of return which equates the net present value of the project's cash flows to zero; or equivalently the rate of return which equates the present value of inflows to the present value of cash outflows. The internal rate of return (IRR) solves the following equation:

$$\sum_t \frac{X_t}{(1+IRR)^t} = 0 \tag{11}$$

In determining whether to accept or reject a particular project, the IRR decision rule is

Accept a project if IRR > r_p
Reject a project if IRR < r_p

Here r_p is the required return on the project. Hence, the IRR rule reverses the logic of the NPV rule. When we compute NPVs, we calculate the NPV for a *given* discount rate on the project, and accept a project whenever the NPV is positive. If we use the IRR rule, we calculate that discount rate that makes the NPV equal to zero. Both methods are related. A typical investment proposal will have cash outflows from capital expenditure at the beginning, followed by cash inflows. Then the NPV is a decreasing function of the

discount rate. Hence, if the NPV is zero for some discount rate, it is positive for all discount rates below that, and negative for all discount rates above this. In this case both methods come to the same conclusion.

We illustrate the use of the IRR rule, and some of the pitfalls of this approach via a series of examples.

Example 3

Suppose a firm whose required rate of return is 10% is considering a project with the following cash flows:

Year	0	1	2	3	4
Cash flow	-1,000	400	400	400	400

Is this a worthwhile investment?

The internal rate of return of this project is the rate of return that solves:

$$\left[\frac{1-(1+IRR)^{-4}}{IRR}\right] \cdot 400 - 1{,}000 = 0 \tag{12}$$

Note that we have to interpolate or use an iterative technique such as Excel's Solver to find the IRR in this case.

The internal rate of return of this project turns out to be 21.86%. Applying the decision rules above, we would accept the project since

$IRR > r_p$ (i.e., 21.86%>10%).

The graph below shows the NPV of this project using various discount rates.

You can see that the NPV of the project decreases as you increase the discount rate. The NPV-function cuts the horizontal axis at the IRR of 21.86% in this case. For all discount rates above 21.86% the NPV of the project is negative, for all discount rates below the IRR the NPV of the project is positive, and since the discount rate is 10%, the project should be accepted. Both decision rules come to the same conclusion.
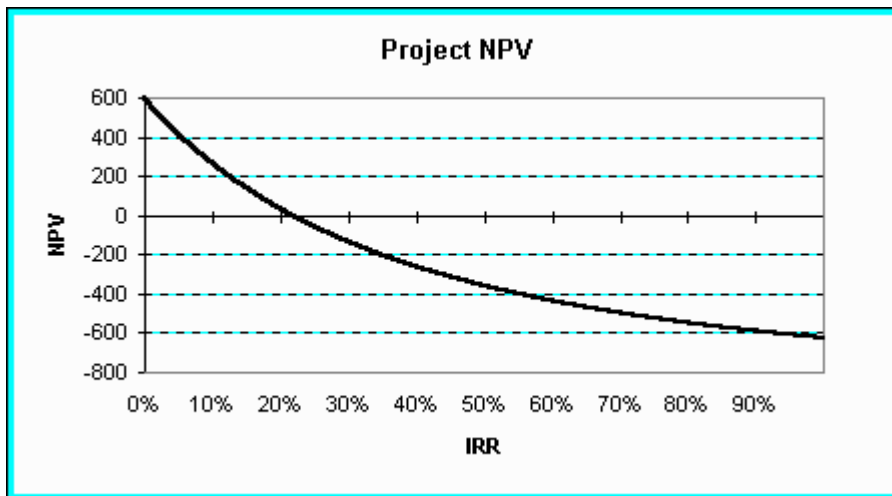
6.8. Problems of the IRR

Several other problems of the internal rate of return are apparent by considering the following table

.Project	0	1	2	IRR	NPV @ 10%	NPV @ 20%
A	-5000	8000	0	60%	2273	1667
B	-5000	0	9800	40%	3099	1806
C	5000		-9800	40%	-3099	-1806
D	-10000	15000	0	50%	3636	2500
E	-5000	16000	-12000	100%, 20%	-372	0

Problem 1: Different Time Horizons

If we compare projects A and B, we see that project A has the higher IRR, whereas B has a higher NPV. Here the IRR fails to recognize the fact that investing money in project B

ensures that we obtain a superior return over a much longer time horizon (two periods), whereas project B gives the high return only for one period. If we use the IRR criterion for choosing between A and B, we assume implicitly that we can reinvest the cash flow of 8000 at the end of period 1 at 60% for another year. The NPV criterion recognizes the fact that we can reinvest the 8000 only at the cost of capital.

Problem 2: Multiple IRRs

The IRR gives us no way to distinguish between projects A, B and E. Project E comes with a huge liability at the end. This can occur, e. g. if a machine has dismantling costs at the end, or if a project requires substantial environmental repairs upon termination (example: open cast mining). Then the equation:

$$NPV = CF_0 + \frac{CF_1}{1+IRR} + \frac{CF_2}{(1+IRR)^2} = -5{,}000 + \frac{16{,}000}{1+IRR} - \frac{12{,}000}{(1+IRR)^2} = 0 \qquad (12)$$

has two solutions: 20% and 100%, hence our criterion fails. This is a consequence of the fact that the sign of the cash flows changes more than once over the lifetime of the project: cash outflows followed by inflows followed by outflows. To make matters worse, if we had a project with a longer horizon than two periods, and the pattern would have N changes of signs, than we would also obtain up to N different IRRs! In this case the IRR criterion is completely worthless.

Problem 3: Different scales

If we compare projects A and D on the basis of their IRRs, we would choose project A over project D, even though project A has a lower NPV. The IRR does not take into account the scale at which we operate these projects. However, if we are not capital constrained we should always invest the project which maximizes wealth, even if it requires a larger capital outlay (and if we were capital constrained, we should pursue a different strategy altogether). There is no reasonable sense in which we can say that project A is more efficient. Project D generates more wealth and should be chosen.

Problem 4: Different signs of cash flows

Projects B and C have exactly the same IRRs, but one has a positive and the other one a negative NPV. The IRR criterion does not account for the fact that with project B we have cash outflows first, and cash inflows later, and the opposite with project C. Effectively, project B is an investment, where we invest money up front in order to receive a return later. Project C is a financing opportunity, where we receive money first and have to repay it later. If we apply the IRR criterion, we basically say "the higher the return the better". However, for financing we want to use the opposite criterion: The lower the IRR, the lower the costs of financing, and the better we are off. This problem is not really an inconsistency, and we can take care of it by modifying the IRR criterion:

- If cash outflows are followed by cash inflows (investments), accept the project if the IRR exceeds the cost of capital (cutoff rate)

- If cash inflows are followed by cash outflows (financing), accept the project if the IRR is lower than the cutoff rate.

6.9 What use is the IRR?

The IRR has one strong attraction: it provides a rate of return which is easier to interpret than the NPV. Hence, are there any applications where we would be able to use it? The answer is: very few. Essentially, we have to be careful that none of the above problems occurs. An example would be a mortgage. A mortgage is a financing, where one cash inflow is followed by a sequence of cash outflows. Hence, the cash flow pattern has only one sign change, hence the IRR is unique (avoids problem 2). Moreover, you need to compare mortgages with the same repayment horizon in order to avoid problem 1, and for the same amount in order avoid problem 3. Then you can use the rule that the mortgage with the lower rate (i. e. the lower IRR) is better. Hence, the moral is that you can use the IRR for some stylized situations, but not for the general capital budgeting problem, where the NPV is the dominant criterion that is robust to the problems listed above.

6.10 Payback Period (PP)

The payback period, PP, is the length of time it takes to recover the initial investment of the project. To apply the payback period criterion, it is necessary for management to establish a maximum acceptable payback value PP*. In practice, PP* is usually between

2 and 4 years. In determining whether to accept or reject a particular project, the payback

period decision rule is:

Accept if PP < PP*
Reject if PP > PP*

For mutually exclusive alternatives accept the project with the lowest PP if PP<PP*

Example 4

Suppose a firm is considering two mutually exclusive projects, C and D, where

the firm's required rate of return is 10% and the projects' cash flows are:

	Project C	Project D
Year 0 Flow	-1,000	-1,000
Year 1 Flow	200	600
Year 2 Flow	800	300
Year 3 Flow	25	1,000
PP	2	3
NPV	-$138	$548

The payback method dictates that project C should be accepted, however the NPV

indicates that if C is accepted, the share price will fall. It appears that the payback

method is not consistent with the goal of shareholder wealth maximization. The

problems with the payback method are that:

- It ignores the time value of money;

- It ignores the cash flows that occur after the payback period; and,

- It ignores the scale of investment.

6.11 Payback Period: accounting for money at risk

One of the attractions of the payback period is that it provides some measure of the

"money at risk". At the start of the project we are presented with a lot of uncertainty

about future cash flows, and the economic environment and our cash flows may turn out more or less favorable than we initially anticipated, with uncertainty being larger for those cash flows in the more distant future. However, the payback criterion is the wrong method to account for that. There are two tools for analyzing the risk associated with more distant cash flows. The first concerns the setting of discount rates. We shall see later that discount rates can be decomposed into a risk-free rate, which is a compensation for the time value of money, and a risk premium, which rewards investors for risk. Hence, the discount rate is:

Discount rate = Risk free rate + Risk premium

Suppose the risk free rate is 10% and the required risk premium is 5% (we will discuss how to determine risk premiums later in the course). Then we obtain the following relationship between the discount rates and the time horizon:

	Period 1	**Period 2**	**Period 3**	**Period 4**
Discount factor at 10%	0.91	0.83	0.75	0.68
Discount factor at 15%	0.87	0.76	0.66	0.57
Difference	0.0395	0.0703	0.0938	0.1113
% Difference	4.35	8.51	12.48	16.29

The picture emerges quite clearly: the risk premium reduces the value of one dollar at the end of period 1 from $0.91 to $0.87, or by 3.95 cents or 4.35%. However, dollars at the end of period 2 are reduced substantially more by 7.03 cents or 8.51%, and the reduction increases with the time horizon. Hence, our NPV criterion, with appropriately set discount rates already accounts for the fact that risk increases with the time horizon.

In addition to this, we know much less about the more distant future than the immediate future, and we would typically change the design of a project if circumstances change in the future. Hence, we need to reflect the fact that one project commits our money for a short period of time and another one for a long period of time in our analysis, because projects with longer time horizons give us somewhat less flexibility. We shall see later in the course that this argument has also some merit, because flexibility has economic value. However, the appropriate tool for analyzing flexibility is decision tree analysis or option analysis (so-called "real options"). We can extend NPV analysis and allow us to quantify the value of flexibility and of "money at risk" by using decision trees and real option analysis. Using payback period is an illegitimate shortcut.

6.12 Profitability Index

Another capital budgeting technique, the profitability index, is used when firms have only a limited supply of capital with which to invest in positive NPV projects. This type of problem is referred to as a capital rationing problem. Given that the objective is to maximize shareholder wealth, the objective in the capital rationing problem is to identify that subset of projects that collectively have the highest aggregate net present value. To assist in that evaluation, this method requires that we compute each project's profitability index PI.

$$PI = \frac{NPV}{I} \tag{13}$$

We then rank the project's PI from highest to lowest and then select from the top of the list until the capital budget is exhausted. The idea behind the profitability index method is that this will provide the subset of projects that maximize the aggregate net present value. However, this is not always the case (as the examples below show).

Example 5

Suppose a firm is considering the following investment projects and has $12,000 available to invest.

Project	Investment Cost	NPV	PI
A	1,000	600	0.6
B	4,000	2,000	0.5
C	6,000	2,400	0.4
D	2,000	400	0.2
E	5,000	500	0.1

In this case, the profitability index is successful since the combination of projects A, B, and C provides the highest aggregate net present value of $5,000. The second best alternative is projects B, C and D with an aggregate net present value of $4,800. Note, however, that if the firm had invested in all of the projects it would have an aggregate net present value of $5,900.

Suppose a firm is considering the following investment projects and has only $12,000 available to invest

Project	Investment Cost	NPV	PI
A	1,000	600	0.6
B	4,000	2,000	0.5
C	6,000	2,400	0.4
D	2,000	700	0.35
E	5,000	500	0.1

In this case, the profitability index does not work. The best alternative is BCD with an aggregate net present value of $5,100. The project ranked most highly by the PI (i.e., Project A) is not even included in the final set. Once again, note that if

the firm had not imposed the capital rationing constraint of $12,000 it could have taken all of the projects and thus achieved an aggregate net present value of $6,200.

In this example the profitability index leads to the wrong conclusion and guides us to make a decision that reduces shareholder wealth. The reason is that we assumed - and this is realistic in many scenarios - that the projects are not divisible: either we commit ourselves to project A or we do not, but we cannot do half of it. In some cases it may however be the case that decisions are scalable: e. g., we can issue $100 million worth of bonds or none, but we can also issue $50million or $75 million. Then the profitability index gives us exactly the right answer:

> Reconsider example 5 above. If project D is divisible, the firm could undertake A, B and C, and half of D. This would give a total NPV of 600 +2000 +2,400 +350 =5350, and this is the maximum the firm can achieve.

> If you know some optimization theory, you may recognize this problem: if the project is divisible, then we maximize the objective subject to a constraint, and this constraint has a so-called shadow-price, and the profitability index is exactly this price. However, if the projects are not divisible, we have in addition to this so-called integer constraints, and we need to take care of these in addition to the rationing constraint.

6.13 Determination of Cash Flows in the NPV analysis

When we examined an investment proposal we were interested only in the marginal or incremental cash flows associated with the project in question. Hence, we will concentrate on the exact determination of the cash flows. The incremental net cash flow

of an investment proposal is defined to be the difference between the firm's cash flows if the investment project is undertaken and the firm's cash flows if the investment project is not undertaken. In this section we show how cash flows are related to accounting numbers and taxes. Define the net cash flow generated by certain assets as:

Net Cash Flow = Cash Inflow - Cash Outflow

$$= \text{Revenue - Expenses - Capital Expenditure - Taxes} \tag{13}$$

The income tax paid is determined by:

$$\text{Taxes} = t*(\text{Revenue - Expenses - Depreciation}) \tag{14}$$

where t is the corporate tax rate. Note that depreciation is not a cash expense and only affects cash flows through its effect on taxes. Substituting equation (13) into equation (14) yields an expression for the firm's cash flow:

$$\text{Cash flow} = (1-t)*(\text{Revenue - Expenses}) + t*\text{Depreciation - Capital expenditure} \tag{15}$$

The term tD_t is sometimes referred to as the *depreciation tax shield*.

Depreciation

With the enactment of the U.S. Tax Reform Act of 1986, depreciable assets generally fall into one of three classes:

- five-year recovery,

- seven-year recovery, and

- real estate.

The five-year recovery class includes (i) automobiles and trucks, (ii) computers and peripheral equipment, calculators, typewriters and copiers, and (iii) items used for research and experimentation. The seven-year recovery class includes books, furniture and office equipment not included in the five-year class.

For items in the five-year and seven-year classes, you can deduct the entire cost in the first year of the item's life, up to $10,000. For items exceeding this amount, there are two choices of depreciation methods - the double declining balance method and the straight-line method. A mid-year convention is followed in the year of acquisition and the year of disposal. Under this convention, a half-year of depreciation is taken in the year of acquisition and the year of disposal. Real estate is depreciated using the straight-line method and a mid-month convention. Residential real estate is 27.5 year property, and non-residential real estate is 31.5 year property.

For the purposes of this course, we will use straight-line depreciation to facilitate computations. Practically speaking, the firm would not elect to use the straight-line method if an accelerated method is allowed. However, straight-line depreciation simplifies the calculations and conveys all the necessary intuition.

There are a number of points to note regarding the depreciation charge:

1. Sales tax, delivery charges and installation are regarded as part of the cost of the equipment and are added to the cost of the asset for depreciation purposes.

2. If the equipment has an estimated salvage value at the end of its useful life, the asset is depreciated down to this value. For example, if the asset costs $20,000 and has an estimated salvage value of $5,000 at the end of its useful life of n years, the annual straight-line depreciation is given by:

$$D = \frac{20,000 - 5,000}{n}$$

Thus, if this asset had a useful life of n = 5 years, then the depreciation charge per year would be:

$$D = \frac{20,000 - 5,000}{n} = \frac{20,000 - 5,000}{5} = \$3,000$$

3. If an asset is sold at an amount in excess of its depreciated (also known as book) value, then this excess is a taxable gain. Similarly, if the asset is sold for an amount less than its depreciated value, then this difference is an allowable deduction for tax purposes.

4. Any costs associated with old equipment, such as removal expenses, do not count as part of the cost of the new machinery and are expensed immediately (i.e., at time 0).

Example 6

A corporation is considering installing a machine that costs $60,000 plus installation costs of $2,000. It will generate revenues of $155,000 annually and cash expenses annually of $100,000. It will be depreciated to a salvage of $6,000 over a seven-year life using the straight-line method. Assuming the firm has a marginal cost of capital of 12 percent and is in the 34 percent marginal tax bracket, determine the incremental cash flows of this investment project. What is the present value of this project?

Year 0: The incremental cash flows associated with the project in year 0 are:

Cost of new machine: $60,000

Installation Cost: $2,000

Years 1-7:

Yearly revenues: $155,000
Yearly expenses: $100,000

Yearly tax expense: t [taxable income] where taxable income = revenues - expenses - depreciation. In this case depreciation is computed using the straight line method, D = (62,000-6,000)/7 = $8,000. Therefore, yearly tax expense is 0.34(155,000-100,000-8,000)=$15,980.

Year 7 salvage value: $6,000.

We can summarize this information in three tables. The first computes taxable income and the tax expense. The second table computes the net cash flow. The third table calculates the project's present value.

Computation of Taxable Income:

Year	Revenues (+)	Expenses (-)	Depreciation(-)	Taxable Income	Tax
1	155,000	100,000	8,000	47,000	15,980
2	155,000	100,000	8,000	47,000	15,980
3	155,000	100,000	8,000	47,000	15,980
4	155,000	100,000	8,000	47,000	15,980
5	155,000	100,000	8,000	47,000	15,980
6	155,000	100,000	8,000	47,000	15,980
7	155,000	100,000	8,000	47,000	15,980

Computation of Net Cash Flows:

Year	Revenues(+)	Expenses (-)	Taxes (-)	Cost (-)	Salvage (+)	Total
0				62,000		-62,000
1	155,000	100,000	15,980			39,020
2	155,000	100,000	15,980			39,020
3	155,000	100,000	15,980			39,020
4	155,000	100,000	15,980			39,020
5	155,000	100,000	15,980			39,020
6	155,000	100,000	15,980			39,020
7	155,000	100,000	15,980		6,000	45,020

Present Value of Project

Year	Cash Flow	Discount Factor @ 12%	Present Value
0	-62,000	1.00	-62,000
1	39,020	1.12	34,839
2	39,020	1.12^2	31,107
3	39,020	1.12^3	27,774
4	39,020	1.12^4	24,798
5	39,020	1.12^5	22,141
6	39,020	1.12^6	19,769
7	45,020	1.12^7	20,365
TOTAL			$118,793

So, the present value of the project is **$118,793**.

Conclusion

In this lecture we discussed the main elements of a methodology to appraise decisions in the firm. The discussion applies to all decisions that change the pattern of future cash flows. We have shown that shareholder value is maximized if and only if the firm makes only those decisions that generate positive net present values. Net present values are calculated by discounting the incremental cash flows of a decision. These may be different from the cash flows that physically relate to a project or investment object. For example, if alternative machines have the same lifetime, than we have to analyze the decision to choose one over the other for a comparable investment horizon.

Alternative criteria are rarely useful. The Payback period and accounting measures are illegitimate shortcuts, and we need to develop proper decision making tools for the problems they try to address. The internal rate of return has a number of serious limitations. Since all criteria are optimal only to the degree that they agree with the NPV rule, it is best to use the NPV rule rather than anything else.

In this lecture we have not given an answer to the question on how to determine the discount rates we have used for project appraisal. We have also not shown how to analyze the riskiness of the cash flows associated with certain decisions. We will address these issues in subsequent lectures.

Important Terminology

Important Formulae

Net Present Value:

$$NPV = \sum_{t=1}^{N} \frac{X_t}{\left(1 + r_p\right)^t} - I$$

Internal Rate of Return:

$$IRR \Rightarrow \sum_{t=1}^{N} \frac{X_t}{\left(1 + IRR\right)^t} - I = 0$$

Profitability Index:

$$PI = \frac{NPV}{I}$$