# How to Win the Stock Market Game 

Developing Short-Term Stock Trading Strategies

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## PART 1

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## I ntroduction

This publication is for short-term traders, i.e. for traders who hold stocks for one to eight days. Short-term trading assumes buying and selling stocks often. After two to four months a trader will have good statistics and he or she can start an analysis of trading results. What are the main questions, which should be answered from this analysis?

- Is my trading strategy profitable?
- Is my trading strategy safe?
- How can I increase the profitability of my strategy and decrease the risk of trading?

No doubt it is better to ask these questions before using any trading strategy. We will consider methods of estimating profitability and risk of trading strategies, optimally dividing trading capital, using stop and limit orders and many other problems related to stock trading.

## Comparison of Trading Strategies

Consider two hypothetical trading strategies. Suppose you use half of your trading capital to buy stocks selected by your secret system and sell them on the next day. The other half of your capital you use to sell short some specific stocks and close positions on the next day.

In the course of one month you make 20 trades using the first method (let us call it strategy \#1) and 20 trades using the second method (strategy \#2). You decide to analyze your trading results and make a table, which shows the returns (in \%) for every trade you made.

| $\#$ | Return per trade in \% <br> Strategy 1 | Return per trade in \% <br> Strategy 2 |
| :---: | :---: | :---: |
| 1 | +3 | +4 |
| 2 | +2 | -5 |
| 3 | +3 | +6 |
| 4 | -5 | +9 |
| 5 | +6 | -16 |
| 6 | +8 | +15 |
| 7 | -9 | +4 |
| 8 | +5 | -19 |
| 9 | +6 | +14 |
| 10 | +9 | +2 |
| 11 | +1 | +9 |
| 12 | -5 | -10 |
| 13 | -2 | +8 |
| 14 | +0 | +15 |
| 15 | -3 | -16 |
| 16 | +4 | +8 |
| 17 | +7 | -9 |
| 18 | +2 | +8 |
| 19 | -4 | +16 |
| 20 | +3 | -5 |

The next figure graphically presents the results of trading for these strategies.


Returns per trades for two hypothetical trading strategies
Which strategy is better and how can the trading capital be divided between these strategies in order to obtain the maximal profit with minimal risk? These are typical trader's questions and we will outline methods of solving them and similar problems.

The first thing you would probably do is calculate of the average return per trade. Adding up the numbers from the columns and dividing the results by 20 (the number of trades) you obtain the average returns per trade for these strategies

```
Rav1 = 1.55%
Rav2 = 1.9%
```

Does this mean that the second strategy is better? No, it does not! The answer is clear if you calculate the total return for this time period. A definition of the total return for any given time period is very simple. If your starting capital is equal to $\mathbf{C 0}$ and after some period of time it becomes C 1 then the total return for this period is equal to

```
Total Return = (C1-C0)/ CO * 100%
```

Surprisingly, you can discover that the total returns for the described results are equal to

```
Total Return1 = 33%
Total Return2 = 29.3%
```

What happened? The average return per trade for the first strategy is smaller but the total return is larger! Many questions immediately arise after this "analysis":

- Can we use the average return per trade to characterize a trading strategy?
- Should we switch to the first strategy?
- How should we divide the trading capital between these strategies?
- How should we use these strategies to obtain the maximum profit with minimal risk?

To answer these questions let us introduce some basic definitions of trading statistics and then outline the solution to these problems.

## Return per Trade

Suppose you bought $N$ shares of a stock at the price $\mathbf{P 0}$ and sold them at the price $\mathbf{P 1}$. Brokerage commissions are equal to COM. When you buy, you paid a cost price

## Cost $=\mathbf{P O} \mathbf{N}+\mathbf{C O M}$

When you sell you receive a sale price
Sale $=\mathbf{P 1} * \mathbf{N}-\mathbf{C O M}$
Your return R for the trade (in \%) is equal to
R = (Sale - Cost)/ Cost * 100\%

## Average Return per Trade

Suppose you made n trades with returns $\mathbf{R 1}, \mathbf{R 2}, \mathbf{R 3}, \ldots, \mathbf{R n}$. One can define an average return per trade Rav
$R a v=(R 1+R 2+R 3+\ldots+R n) / n$

This calculations can be easily performed using any spreadsheet such as MS Excel, Origin, ... .

## More about average return

You can easily check that the described definition of the average return is not perfect. Let us consider a simple case.

Suppose you made two trades. In the first trade you have gained $50 \%$ and in the second trade you have lost $50 \%$. Using described definition you can find that the average return is equal to zero. In practice you have lost $25 \%$ ! Let us consider this contradiction in details.

Suppose your starting capital is equal to $\$ 100$. After the first trade you made $50 \%$ and your capital became

```
$100* 1.5 = $150
```

After the second trade when you lost 50\% your capital became

## $\$ 150 * 0.5=\$ 75$

So you have lost $\$ 25$, which is equal to $-25 \%$. It seems that the average return is equal to - $25 \%$, not 0\%.

This contradiction reflects the fact that you used all your money for every trade. If after the first trade you had withdrawn $\$ 50$ (your profit) and used $\$ 100$ (not $\$ 150$ ) for the second trade you would have lost $\$ 50$ (not $\$ 75$ ) and the average return would have been zero.

In the case when you start trading with a loss (\$50) and you add $\$ 50$ to your trading account and you gain $50 \%$ in the second trade the average return will be equal to zero. To use this trading method you should have some cash reserve so as to an spend equal amount of money in every trade to buy stocks. It is a good idea to use a part of your margin for this reserve.

However, very few traders use this system for trading. What can we do when a trader uses all his trading capital to buy stocks every day? How can we estimate the average return per trade?

In this case one needs to consider the concept of growth coefficients.

## Growth Coefficient

Suppose a trader made n trades. For trade \#1

## K1 = Sale1 / Cost1

where Sale1 and Costl represent the sale and cost of trade \#1. This ratio we call the growth coefficient. If the growth coefficient is larger than one you are a winner. If the growth coefficient is less than one you are a loser in the given trade.

If K1, K2, ... are the growth coefficients for trade \#1, trade \#2, ... then the total growth coefficient can be written as a product

## $K=K 1 * K 2 * K 3 *$...

In our previous example the growth coefficient for the first trade K1 = $\mathbf{1 . 5}$ and for the second trade $\mathbf{K 2} \mathbf{=} \mathbf{0 . 5}$. The total growth coefficient, which reflects the change of your trading capital is equal to

```
K=1.5* 0.5 = 0.75
```

which correctly corresponds to the real change of the trading capital. For $n$ trades you can calculate the average growth coefficient Kav per trade as

Kav $=($ K1*K2*K3*...) ^(1/n)
These calculations can be easily performed by using any scientific calculator. The total growth coefficient for $n$ trades can be calculated as
$K=K a v{ }^{\wedge} n$
In our example $\operatorname{Kav}=(\mathbf{1 . 5} \boldsymbol{*} \mathbf{0 . 5})^{\wedge} \mathbf{1 / 2} \mathbf{2}=\mathbf{0 . 8 6 6}$, which is less than 1 . It is easily to check that

### 0.866 ^ $2=0.866 * 0.866=0.75$

However, the average returns per trade Rav can be used to characterize the trading strategies. Why? Because for small profits and losses the results of using the growth coefficients and the average returns are close to each other. As an example let us consider a set of trades with returns

$$
\begin{gathered}
\hline \hline R 1=-5 \% \\
R 2=+7 \% \\
R 3=-1 \% \\
R 4=+2 \% \\
R 5=-3 \% \\
R 6=+5 \% \\
R 7=+0 \% \\
R 8=+2 \% \\
R 9=-10 \% \\
R 10=+11 \% \\
R 11=-2 \% \\
R 12=5 \% \\
R 13=+3 \% \\
R 14=-1 \% \\
R 15=2 \% \\
\hline
\end{gathered}
$$

The average return is equal to

$$
\operatorname{Rav}=(-5+7-1+2-3+5+0+2-10+11-2+5+3-1+2) / 15=+1 \%
$$

The average growth coefficient is equal to

```
Kav=(0.95*1.07*0.99*1.02*0.97*1.05*1*1.02*0.9*1.11*0.98*1.05*1.03*0.99*1.02)^(1/15) = 1.009
```

which corresponds to $\mathbf{0 . 9 \%}$. This is very close to the calculated value of the average return = $\mathbf{1 \%}$. So, one can use the average return per trade if the return per trades are small.

Let us return to the analysis of two trading strategies described previously. Using the definition of the average growth coefficient one can obtain that for these strategies

```
Kav1 \(=1.014\)
\(K a v 2=1.013\)
```

So, the average growth coefficient is less for the second strategy and this is the reason why the total return using this strategy is less.

## Distribution of returns

If the number of trades is large it is a good idea to analyze the trading performance by using a histogram. Histogram (or bar diagram) shows the number of trades falling in a given interval of returns. A histogram for returns per trade for one of our trading strategies is shown in the next figure


Histogram of returns per trades for the Low Risk Trading Strategy
As an example, we have considered distribution of returns for our Low Risk Trading Strategy (see more details in http://www.stta-consulting.com) from January 1996 to April 2000. The bars represent the number of trades for given interval of returns. The largest bar represents the number of trades with returns between 0 and $5 \%$. Other numbers are shown in the Table.

| Return Range, \% | Number of Stocks | Return Range, \% | Number of Stocks |
| :---: | :---: | :---: | :---: |
| $0<R<5$ | $\mathbf{2 4 9}$ | $-5<R<0$ | $\mathbf{1 7 1}$ |
| $5<R<10$ | $\mathbf{1 7 4}$ | $-10<R<-5$ | $\mathbf{8 5}$ |
| $10<R<15$ | $\mathbf{1 2 7}$ | $-15<R<-10$ | $\mathbf{4 6}$ |
| $15<R<20$ | $\mathbf{7 2}$ | $-20<R<-15$ | $\mathbf{1 7}$ |
| $20<R<25$ | $\mathbf{4 7}$ | $-25<R<-20$ | $\mathbf{5}$ |
| $25<R<30$ | $\mathbf{2 5}$ | $-30<R<-25$ | $\mathbf{6}$ |
| $30<R<35$ | $\mathbf{1 7}$ | $-35<R<-30$ | $\mathbf{1}$ |
| $35<R<40$ | $\mathbf{4}$ | $-40<R<-35$ | $\mathbf{3}$ |

For this distribution the average return per trade is $4.76 \%$. The width of histogram is related to a very important statistical characteristic: the standard deviation or risk.

## Risk of trading

To calculate the standard deviation one can use the equation

$$
s=\sqrt{\frac{1}{n-1} \sum_{i=1}^{n}(R i-R a v)^{2}}
$$

The larger the standard deviation, the wider the distribution of returns. A wider distribution increases the probability of negative returns, as shown in the next figure.



Distributions of returns per trade for Rav = 3\% and for different standard deviations
Therefore, one can conclude that a wider distribution is related to a higher risk of trading. This is why the standard distribution of returns is called the risk of trading. One can also say that risk is a characteristic of volatility of returns.

An important characteristic of any trading strategy is

## Risk-to-Return Ratio = s/ Rav

The smaller the risk-to-return ratio, the better the trading strategy. If this ratio is less than 3 one can say that a trading strategy is very good. We would avoid any trading strategy for which the risk-to-return ration is larger than 5. For distribution in Fig. 1.2 the risk-to-return ratio is equal to 2.6, which indicates low level of risk for the considered strategy.

Returning back to our hypothetical trading strategies one can estimate the risk to return ratios for these strategies. For the first strategy this ratio is equal to 3.2. For the second strategy it is equal to 5.9. It is clear that the second strategy is extremely risky, and the portion of trading capital for using this strategy should be very small.

How small? This question will be answered when we will consider the theory of trading portfolio.

## More about risk of trading

The definition of risk introduced in the previous section is the simplest possible. It was based on using the average return per trade. This method is straightforward and for many cases it is sufficient for comparing different trading strategies.

However, we have mentioned that this method can give false results if returns per trade have a high volatility (risk). One can easily see that the larger the risk, the larger the difference between estimated total returns using average returns per trade or the average growth coefficients. Therefore, for highly volatile trading strategies one should use the growth coefficients K.

Using the growth coefficients is simple when traders buy and sell stocks every day. Some strategies assume specific stock selections and there are many days when traders wait for opportunities by just watching the market. The number of stocks that should be bought is not constant.

In this case comparison of the average returns per trade contains very little information because the number of trades for the strategies is different and the annual returns will be also different even for equal average returns per trade.

One of the solutions to this problem is considering returns for a longer period of time. One month, for example. The only disadvantage of this method is the longer period of time required to collect good statistics.

Another problem is defining the risk when using the growth coefficients. Mathematical calculation become very complicated and it is beyond the topic of this publication. If you feel strong in math you can write us (service@stta-consulting.com) and we will recommend you some reading about this topic. Here, we will use a tried and true definition of risk via standard deviations of returns per trade in \%. In most cases this approach is sufficient for comparing trading strategies. If we feel that some calculations require the growth coefficients we will use them and we will insert some comments about estimation of risk.

The main goal of this section to remind you that using average return per trade can slightly overestimate the total returns and this overestimation is larger for more volatile trading strategies.

## Correlation Coefficient

Before starting a description of how to build an efficient trading portfolio we need to introduce a new parameter: correlation coefficient. Let us start with a simple example.

Suppose you trade stocks using the following strategy. You buy stocks every week on Monday using your secret selection system and sell them on Friday. During a week the stock market (SP 500 Index) can go up or down. After 3 month of trading you find that your result are strongly correlated with the market performance. You have excellent returns for week when the market is up and you are a loser when market goes down. You decide to describe this correlation mathematically. How to do this?

You need to place your weekly returns in a spreadsheet together with the change of SP 500 during this week. You can get something like this:

| Weekly Return, \% | Change of SP 500, \% |
| :---: | :---: |
| 13 | 1 |
| -5 | -3 |
| 16 | 1 |
| 4 | 3.2 |
| 20 | 5 |
| 21 | 5.6 |
| -9 | -3 |
| -8 | -1.2 |
| 2 | -1 |
| 8 | 6 |
| 7 | -2 |
| 26 | 3 |

These data can be presented graphically.


## Dependence of weekly returns on the SP 500 change for hypothetical strategy

Using any graphical program you can plot the dependence of weekly returns on the SP 500 change and using a linear fitting program draw the fitting line as in shown in Figure. The correlation coefficient $\mathbf{c}$ is the parameter for quantitative description of deviations of data points from the fitting line. The range of change of $c$ is from -1 to +1 . The larger the scattering of the points about the fitting curve the smaller the correlation coefficient.

The correlation coefficient is positive when positive change of some parameter (SP 500 change in our example) corresponds to positive change of the other parameter (weekly returns in our case).

The equation for calculating the correlation coefficient can be written as

$$
C=\frac{\sum_{i=1}^{N}\left(X_{i}-X_{a v}\right)\left(Y_{i}-Y_{a v}\right)}{S_{x} S_{y}}
$$

where $\mathbf{X}$ and $\mathbf{Y}$ are some random variable (returns as an example); $\mathbf{S}$ are the standard deviations of the corresponding set of returns; $\mathbf{N}$ is the number of points in the data set.

For our example the correlation coefficient is equal to 0.71 . This correlation is very high. Usually the correlation coefficients are falling in the range (-0.1, 0.2).

We have to note that to correctly calculate the correlation coefficients of trading returns one needs to compare $\mathbf{X}$ and $\mathbf{Y}$ for the same period of time. If a trader buys and sells stocks every day he can compare daily returns (calculated for the same days) for different strategies. If a trader buys stocks and sells them in 2-3 days he can consider weekly or monthly returns.

Correlation coefficients are very important for the market analysis. Many stocks have very high correlations. As an example let us present the correlation between one days price changes of MSFT and INTC.


## Correlation between one days price change of I NTC and MSFT

The presented data are gathered from the 1988 to 1999 year period. The correlation coefficient $\mathbf{c}=\mathbf{0 . 3 6 1}$, which is very high for one day price change correlation. It reflects simultaneous buying and selling these stocks by mutual fund traders.

Note that correlation depends on time frame. The next Figure shows the correlation between ten days (two weeks) price changes of MSFT and INTC.


## Correlation between ten day price change of I NTC and MSFT

The ten day price change correlation is slightly weaker than the one day price change correlation. The calculation correlation coefficient is equal to 0.327 .

## Efficient Trading Portfolio

The theory of efficient portfolio was developed by Harry Markowitz in 1952. (H.M.Markowitz, "Portfolio Selection," Journal of Finance, 7, 77 - 91, 1952.) Markowitz considered portfolio diversification and showed how an investor can reduce the risk of investment by intelligently dividing investment capital.

Let us outline the main ideas of Markowitz's theory and tray to apply this theory to trading portfolio. Consider a simple example. Suppose, you use two trading strategies. The average daily returns of these strategies are equal to $\mathbf{R 1}$ and $\mathbf{R 2}$. The standard deviations of these returns (risks) are s1 and s2. Let q1 and $\mathbf{q 2}$ be parts of your capital using these strategies.
$\mathrm{q} 1+\mathrm{q} 2=1$

## Problem:

Find q 1 and q 2 to minimize risk of trading.

## Solution:

Using the theory of probabilities one can show that the average daily return for this portfolio is equal to

## $\mathbf{R}=\mathbf{q} \mathbf{1 *} \mathbf{R 1}+\mathbf{q} \mathbf{2} * \mathbf{R} \mathbf{2}$

The squared standard deviation (variance) of the average return can be calculated from the equation

$$
s^{2}=(q 1 * s 1)^{2}+(q 2 * s 2)^{2}+2 * c * q 1 * s 1 * q 2 * s 2
$$

where $\mathbf{c}$ is the correlation coefficient for the returns $\mathbf{R 1}$ and $\mathbf{R 2}$.
To solve this problem it is good idea to draw the graph $\mathbf{R}, \mathbf{s}$ for different values of $\mathbf{q} \mathbf{1}$. As an example consider the two strategies described in Section 2. The daily returns (calculated from the growth coefficients) and risks for these strategies are equal to

R1 = 1.4\% $\quad$ s1 = 5.0\%
R2 $=1.3 \% \quad$ s2 $=11.2 \%$
The correlation coefficient for these returns is equal to
$\mathrm{c}=0.09$
The next figure shows the return-risk plot for different values of $\mathbf{q 1}$.


Return-Risk plot for the trading portfolio described in the text
This plot shows the answer to the problem. The risk is minimal if the part of trading capital used to buy the first stock from the list is equal to 0.86 . The risk is equal to 4.7 , which is less than for the strategy when the whole capital is employed using the first trading strategy only.

So, the trading portfolio, which provides the minimal risk, should be divided between the two strategies. $86 \%$ of the capital should be used for the first strategy and the $14 \%$ of the capital must be used for the second strategy. The expected return for this portfolio is smaller than maximal expected value, and the trader can adjust his holdings depending on how much risk he can afford. People, who like getting rich quickly, can use the first strategy only. If you want a more peaceful life you can use $\mathbf{q 1}=0.86$ and $\mathbf{q 2}=0.14$, i.e. about $1 / 6$ of your trading capital should be used for the second strategy.

This is the main idea of building portfolio depending on risk. If you trade more securities the Return-Risk plot becomes more complicated. It is not a single line but a complicated figure. Special computer methods of analysis of such plots have been developed. In our publication, we consider some simple cases only to demonstrate the general ideas.

We have to note that the absolute value of risk is not a good characteristic of trading strategy. It is more important to study the risk to return ratios. Minimal value of this ratio is the main criterion of the best strategy. In this example the minimum of the risk to return ratio is also the value $\mathbf{q 1}=0.86$. But this is not always true. The next example is an illustration of this statement.

Let us consider a case when a trader uses two strategies (\#1 and \#2) with returns and risks, which are equal to
$\begin{array}{ll}\text { R1 }=3.55 \% & s 1=11.6 \% \\ R 2=2.94 \% & s 2=9.9 \%\end{array}$

The correlation coefficient for the returns is equal to

## $\mathrm{c}=0.165$

This is a practical example related to using our Basic Trading Strategy (look for details at http://www.stta-consulting.com).

We calculated return $\mathbf{R}$ and standard deviation $\mathbf{s}$ (risk) for various values of $\mathbf{q 1}$ - part of the capital employed for purchase using the first strategy. The next figure shows the return risk plot for various values of q1.


## Return - risk plot for various values of q1 for strategy described in the text

You can see that minimal risk is observed when $\mathbf{q 1}=0.4$, i.e. $40 \%$ of trading capital should be spend for strategy \#1.

Let us plot the risk to return ratio as a function of $\mathbf{q 1}$.


The risk to return ratio as a function of q1 for strategy described in the text
You can see that the minimum of the risk to return ratio one can observe when $\mathbf{q 1}=$ 0.47 , not 0.4 . At this value of q 1 the risk to return ratio is almost $40 \%$ less than the ratio in the case where the whole capital is employed using only one strategy. In our opinion, this is the optimal distribution of the trading capital between these two strategies. In the table we show the returns, risks and risk to return ratios for strategy \#1, \#2 and for efficient trading portfolio with minimal risk to return ratio.

|  | Average return, \% | Risk, \% | Risk/Return |
| :---: | :---: | :---: | :---: |
| Strategy \#1 | 3.55 | 11.6 | 3.27 |
| Strategy \#2 | 2.94 | 9.9 | 3.37 |
| Efficient Portfolio <br> q1 $=47 \%$ | 3.2 | 8.2 | 2.5 |

One can see that using the optimal distribution of the trading capital slightly reduces the average returns and substantially reduces the risk to return ratio.

Sometimes a trader encounters the problem of estimating the correlation coefficient for two strategies. It happens when a trader buys stocks randomly. It is not possible to construct a table of returns with exact correspondence of returns of the first and the second strategy. One day he buys stocks following the first strategy and does not buy stocks following the second strategy. In this case the correlation coefficient cannot be calculated using the equation shown above. This definition is only true for simultaneous stock purchasing. What can we do in this case? One solution is to consider a longer period of time, as we mentioned before. However, a simple estimation can be performed even for a short period of time. This problem will be considered in the next section.

## PART 2

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## Efficient portfolio and the correlation coefficient.

It is relatively easily to calculate the average returns and the risk for any strategy when a trader has made 40 and more trades. If a trader uses two strategies he might be interested in calculating optimal distribution of the capital between these strategies. We have mentioned that to correctly use the theory of efficient portfolio one needs to know the average returns, risks (standard deviations) and the correlation coefficient. We also mentioned that calculating the correlation coefficient can be difficult and sometimes impossible when a trader uses a strategy that allows buying and selling of stocks randomly, i.e. the purchases and sales can be made on different days.

The next table shows an example of such strategies. It is supposed that the trader buys and sells the stocks in the course of one day.

| Date | Return per purchase <br> for Strategy \#1 | Return per purchase <br> for Strategy \#2 |
| :---: | :---: | :---: |
| Jan 3 | $+5.5 \%$ | $-3.5 \%$ |
| Jan 4 | $+2.5 \%$ |  |
| Jan 5 |  | $-5 \%$ |
| Jan 6 | $-3.2 \%$ |  |
| Jan 7 |  |  |
| Jan 10 | $+1.1 \%$ | $+8 \%$ |
| Jan 11 |  | $9.5 \%$ |
| Jan 12 | $+15.0 \%$ |  |
| Jan 13 | $-7.6 \%$ |  |
| Jan 14 |  | $-5.4 \%$ |

In this example there are only two returns (Jan 3, Jan 10), which can be compared and be used for calculating the correlation coefficient.

Here we will consider the influence of correlation coefficients on the calculation of the efficient portfolio. As an example, consider two trading strategies (\#1 and \#2) with returns and risks:

R1 = $3.55 \% \quad s 1=11.6 \%$
R2 $=2.94$ \% $\quad \mathbf{s 2}=9.9 \%$
Suppose that the correlation coefficient is unknown. Our practice shows that the correlation coefficients are usually small and their absolute values are less than 0.15 .

Let us consider three cases with $\mathbf{c}=-0.15, \mathbf{c}=0$ and $\mathbf{c}=0.15$. We calculated returns $\mathbf{R}$ and standard deviations $\mathbf{S}$ (risk) for various values of $\mathbf{q 1}$ - part of the capital used for purchase of the first strategy. The next figure shows the risk/return plot as a function of $\mathbf{q 1}$ for various values of the correlation coefficient.


## Return - risk plot for various values of q1 and the correlation coefficients for the strategies described in the text.

One can see that the minimum of the graphs are very close to each other. The next table shows the results.

| $\mathbf{c}$ | $\mathbf{q 1}$ | $\mathbf{R}$ | $\mathbf{S}$ | $\mathbf{S / R}$ |
| :---: | :---: | :---: | :---: | :---: |
| -0.15 | 0.55 | 3.28 | 7.69 | 2.35 |
| 0 | 0.56 | 3.28 | 8.34 | 2.57 |
| 0.15 | 0.58 | 3.29 | 8.96 | 2.72 |

As one might expect, the values of "efficient" returns R are also close to each other, but the risks $S$ depend on the correlation coefficient substantially. One can observe the lowest risk for negative values of the correlation coefficient.

## Conclusion:

The composition of the efficient portfolio does not substantially depend on the correlation coefficients if they are small. Negative correlation coefficients yield less risk than positive ones.

One can obtain negative correlation coefficients using, for example, two "opposite strategies": buying long and selling short. If a trader has a good stock selection system for these strategies he can obtain a good average return with smaller risk.

## Probability of 50\% capital drop

How safe is stock trading? Can you lose more than $50 \%$ of your trading capital trading stocks? Is it possible to find a strategy with low probability of such disaster?

Unfortunately, a trader can lose 50 and more percent using any authentic trading strategy. The general rule is quite simple: the larger your average profit per trade, the large the probability of losing a large part of your trading capital. We will try to develop some methods, which allow you to reduce the probability of large losses, but there is no way to make this probability equal to zero.

If a trader loses $50 \%$ of his capital it can be a real disaster. If he or she starts spending a small amount of money for buying stocks, the brokerage commissions can play a very significant role. As the percentage allotted to commissions increases, the total return suffers. It can be quite difficult for the trader to return to his initial level of trading capital.

Let us start by analyzing the simplest possible strategy.

## Problem:

Suppose a trader buys one stock every day and his daily average return is equal to R. The standard deviation of these returns (risk) is equal to $s$. What is the probability of losing 50 or more percent of the initial trading capital in the course of one year?

## Solution:

Suppose that during one given year a trader makes about 250 trades. Suppose also that the distribution of return can be described by gaussian curve. (Generally this is not true. For a good strategy the distribution is not symmetric and the right wing of the distribution curve is higher than the left wing. However this approximation is good enough for purposes of comparing different trading strategies and estimating the probabilities of the large losses.)

We will not present the equation that allows these calculations to be performed. It is a standard problem from game theory. As always you can write us to find out more about this problem. Here we will present the result of the calculations. One thing we do have to note: we use the growth coefficients to calculate the annual return and the probability of large drops in the trading capital.

The next figure shows the results of calculating these probabilities (in \%) for different values of the average returns and risk-to-return ratios.


## The probabilities (in \%) of 50\% drops in the trading capital for different values of average returns and risk-to-return ratios

One can see that for risk to return ratios less than 4 the probability of losing $50 \%$ of the trading capital is very small. For risk/return $>5$ this probability is high. The probability is higher for the larger values of the average returns.

## Conclusion:

A trader should avoid strategies with large values of average returns if the risk to return ratios for these strategies are larger than 5.

## I nfluence of Commissions

We have mentioned that when a trader is losing his capital the situation becomes worse and worse because the influence of the brokerage commissions becomes larger.

As an example consider a trading method, which yields $2 \%$ return per day and commissions are equal to $1 \%$ of initial trading capital. If the capital drops as much as $50 \%$ then commissions become $2 \%$ and the trading system stops working because the average return per day becomes 0\%.

We have calculated the probabilities of a $50 \%$ capital drop for this case for different values of risk to return ratios. To compare the data obtained we have also calculated the probabilities of $50 \%$ capital drop for an average daily return $=1 \%$ (no commissions have been considered).

For initial trading capital the returns of these strategies are equal but the first strategy becomes worse when the capital becomes smaller than its initial value and becomes better when the capital becomes larger than the initial capital. Mathematically the return can be written as
$\mathbf{R}=\mathbf{R o}$ - commissions/ capital * 100\%
where $\mathbf{R}$ is a real return and $\mathbf{R o}$ is a return without commissions. The next figure shows the results of calculations.


The probabilities (in \%) of 50\% drops in the trading capital for different values of the average returns and risk-to-return ratios. Filled symbols show the case when commissions/ (initial capital) $=1 \%$ and $\mathbf{R o}=2 \%$. Open symbols show the case when $\mathbf{R o}=$ $1 \%$ and commissions $=0$.

One can see from this figure that taking into account the brokerage commissions substantially increases the probability of a $50 \%$ capital drop. For considered case the strategy even with risk to return ratio $=4$ is very dangerous. The probability of losing $50 \%$ of the trading capital is larger than $20 \%$ when the risk of return ratios are more than 4.

Let us consider a more realistic case. Suppose one trader has $\$ 10,000$ for trading and a second trader has $\$ 5,000$. The round trip commissions are equal to $\$ 20$. This is $0.2 \%$ of the initial capital for the first trader and $0.4 \%$ for the second trader. Both traders use a strategy with the average daily return $=0.7 \%$. What are the probabilities of losing $50 \%$ of the trading capital for these traders depending on the risk to return ratios?

The answer is illustrated in the next figure.


The probabilities (in \%) of $50 \%$ drops in the trading capital for different values of the average returns and risk-to-return ratios. Open symbols represent the first trader (\$10,000 trading capital). Filled symbols represent the second trader ( $\$ 5,000$ trading capital). See details in the text.

From the figure one can see the increase in the probabilities of losing $50 \%$ of the trading capital for smaller capital. For risk to return ratios greater than 5 these probabilities become very large for small trading capitals.

Once again: avoid trading strategies with risk to return ratios > 5 .

## Distributions of Annual Returns

Is everything truly bad if the risk to return ratio is large? No, it is not. For large values of risk to return ratios a trader has a chance to be a lucky winner. The larger the risk to return ratio, the broader the distribution of annual returns or annual capital growth.

## Annual capital growth $=($ Capital after 1 year) $/(\operatorname{Initial}$ Capital) $)$

We calculated the distribution of the annual capital growths for the strategy with the average daily return $=0.7 \%$ and the brokerage commissions $=\$ 20$. The initial trading capital was supposed $=\$ 5,000$. The results of calculations are shown in the next figure for two values of the risk to return ratios.


## Distributions of annual capital growths for the strategy described in the text

The average annual capital growths are equal in both cases ( 3.0 or $200 \%$ ) but the distributions are very different. One can see that for a risk to return ratio $=6$ the chance of a large loss of capital is much larger than for the risk to return ratio $=3$. However, the chance of annual gain larger than 10 ( $>900 \%$ ) is much greater. This strategy is good for traders who like risk and can afford losing the whole capital to have a chance to be a big winner. It is like a lottery with a much larger probability of being a winner.

## When to give up

In the previous section we calculated the annual capital growth and supposed that the trader did not stop trading even when his capital had become less than $50 \%$. This makes sense only in the case when the influence of brokerage commissions is small even for reduced capital and the trading strategy is still working well. Let us analyze the strategy of the previous section in detail.

The brokerage commissions were supposed $=\$ 20$, which is $0.4 \%$ for the capital $=$ $\$ 5,000$ and $0.8 \%$ for the capital $=\$ 2,500$.

So, after a $50 \%$ drop the strategy for a small capital becomes unprofitable because the average return is equal to $0.7 \%$. For a risk to return ratio $=6$ the probability of touching the $50 \%$ level is equal to $16.5 \%$. After touching the $50 \%$ level a trader should give up, switch to more profitable strategy, or add money for trading. The chance of winning with the amount of capital $=\$ 2,500$ is very small.

The next figure shows the distribution of the annual capital growths after touching the $50 \%$ level.


## Distribution of the annual capital growths after touching the 50\% level. Initial capital = \$5,000; commissions = \$20; risk/ return ratio = 6; average daily return = 0.7\%

One can see that the chance of losing the entire capital is quite high. The average annual capital growth after touching the $50 \%$ level $(\$ 2,500)$ is equal to 0.39 or $\$ 1950$. Therefore, after touching the $50 \%$ level the trader will lose more money by the end of the year.

The situation is completely different when the trader started with $\$ 10,000$. The next figure shows the distribution of the annual capital growths after touching the $50 \%$ level in this more favorable case.


## Distribution of the annual capital growths after touching the 50\% level. I nitial capital = $\mathbf{\$ 1 0 , 0 0 0 ; ~ c o m m i s s i o n s ~ = ~} \mathbf{\$ 2 0}$; risk/ return ratio = 6; average daily return $=\mathbf{0 . 7 \%}$

One can see that there is a good chance of finishing the year with a zero or even positive result. At least the chance of retaining more than $50 \%$ of the original trading capital is much larger than the chance of losing the rest of money by the end of the year. The average annual capital growth after touching the $50 \%$ level is equal to 0.83 . Therefore, after touching the $50 \%$ level the trader will compensate for some losses by the end of the year.

## Conclusion:

Do not give up after losing a large portion of your trading capital if your strategy is still profitable.

## Cash Reserve

We have mentioned that after a large capital drop a trader can start thinking about using his of her reserve capital. This makes sense when the strategy is profitable and adding reserve capital can help to fight the larger contribution of the brokerage commissions. As an example we consider the situation described in the previous section. Let us write again some parameters:
Initial trading capital $=\$ 5,000$
Average daily return $=0.7 \%$ (without commissions)
Brokerage commissions $=\$ 20$ (roundtrip)
Risk/Return = 3
Reserve capital $=\$ 2,500$ will be added if the main trading capital drops more than $50 \%$.
The next figure shows the distribution of the annual capital growths for this trading method.


Distribution of the annual capital growths after touching the 50\% level. Initial capital $=\$ 5,000 ;$ commissions $=\$ 20$; risk/ return ratio $=6$; average daily return $=0.7 \%$. Reserve capital of $\$ 2,500$ has been used after the $50 \%$ drop of the initial capital

The average annual capital growth after touching the $50 \%$ level for this trading method is equal to 1.63 or $\$ 8,150$, which is larger than $\$ 7,500(\$ 5,000+\$ 2,500)$. Therefore, using reserve trading capital can help to compensate some losses after a $50 \%$ capital drop.

Let's consider a important practical problem. We were talking about using reserve capital $(\$ 2,500)$ only in the case when the main capital $(\$ 5,000)$ drops more than $50 \%$. What will happen if we use the reserve from the very beginning, i.e. we will use $\$ 7,500$ for trading without any cash reserve? Will the average annual return be larger in this case?

Yes, it will. Let us show the results of calculations.
If a trader uses $\$ 5,000$ as his main capital and adds $\$ 2,500$ if the capital drops more than $50 \%$ then in one year he will have on average $\$ 15,100$.

If a trader used $\$ 7,500$ from the beginning this figure will be transformed to $\$ 29,340$, which is almost two times larger than for the first method of trading.

If commissions do not play any role the difference between these two methods is smaller.

As an example consider the described methods in the case of zero commissions. Suppose that the average daily return is equal to $0.7 \%$. In this case using $\$ 5,000$ and $\$ 2,500$ as a reserve yields an average of $\$ 28,000$. Using the entire $\$ 7,500$ yields $\$ 43,000$. This is about a $30 \%$ difference.

Therefore, if a trader has a winning strategy it is better to use all capital for trading than to keep some cash for reserve. This becomes even more important when brokerage commissions play a substantial role.

You can say that this conclusion is in contradiction to our previous statement, where we said how good it is to have a cash reserve to add to the trading capital when the latter drops to some critical level.

The answer is simple. If a trader is sure that a strategy is profitable then it is better to use the entire trading capital to buy stocks utilizing this strategy.

However, there are many situations when a trader is not sure about the profitability of a given strategy. He might start trading using a new strategy and after some time he decides to put more money into playing this game.

This is a typical case when cash reserve can be very useful for increasing trading capital, particularly when the trading capital drops to a critical level as the brokerage commissions start playing a substantial role.

The reader might ask us again: if the trading capital drops why should we put more money into playing losing game? You can find the answer to this question in the next section.

## I s your strategy profitable?

Suppose a trader makes 20 trades using some strategy and loses $5 \%$ of his capital. Does it mean that the strategy is bad? No, not necessarily. This problem is related to the determination of the average return per trade. Let us consider an important example.

The next figure represents the returns on 20 hypothetical trades.


## Bar graph of the $\mathbf{2 0}$ returns per trade described in the text

Using growth coefficients we calculated the total return, which is determined by

```
total return = (current capital - initial capital) / (initial capital) * 100%
```

For the considered case the total return is negative and is equal to $-5 \%$. We have calculated this number using the growth coefficients. The calculated average return per trade is
also negative, and it is equal to $-0.1 \%$ with the standard deviation (risk) $=5.4 \%$. The average growth coefficient is less than 1, which also indicates the average loss per trade.

Should the trader abandon this strategy?
The answer is no. The strategy seems to be profitable and a trader should continue using it. Using the equations presented in part 1 of this publication gives the wrong answer and can lead to the wrong conclusion. To understand this statement let us consider the distribution of the returns per trade.

Usually this distribution is asymmetric. The right wing of the distribution is higher than the left one. This is related to natural limit of losses: you cannot lose more than $100 \%$. However, let us for simplicity consider the symmetry distribution, which can be described by the gaussian curve. This distribution is also called a normal distribution and it is presented in the next figure.


Normal distribution. $s$ is the standard deviation
The standard deviation $\mathbf{s}$ of this distribution (risk) characterizes the width of the curve. If one cuts the central part of the normal distribution with the width $2 \mathbf{s}$ then the probability of finding an event (return per trade in our case) within these limits is equal to $67 \%$. The probability of finding a return per trade within the $4 \mathbf{s}$ limits is equal to $95 \%$.

Therefore, the probability to find the trades with positive or negative returns, which are out of $4 \mathbf{s}$ limits is equal to $5 \%$.

## Lower limit = average return - 2s <br> Upper limit = average return - 2s

The return on the last trade of our example is equal to $-20 \%$. It is out of 2 s and even 4 s limits. The probability of such losses is very low and considering -20\% loss in the same way as other returns would be a mistake.

What can be done? Completely neglecting this negative return would also be a mistake. This trade should be considered separately.

There are many ways to recalculate the average return for given strategy. Consider a simplest case, one where the large negative return has occurred on a day when the market drop is more than $5 \%$. Such events are very rare. One can find such drops one or two times per year. We can assume that the probability of such drops is about $1 / 100$, not $1 / 20$ as for other returns. In this case the average return can be calculated as

Rav $=0.99$ R1 + 0.01 R2
where $\mathbf{R 1}$ is the average return calculated for the first 19 trades and $\mathbf{R 2}=-20 \%$ is the return for the last trade related to the large market drop. In our example R1 $=1 \%$ and $\mathbf{R a v}=0.79 \%$. The standard deviation can be left equal to $5.4 \%$.

This method of calculating the average returns is not mathematically perfect but it reflects real situations in the market and can be used for crude estimations of average returns.

Therefore, one can consider this strategy as profitable and despite loss of some money it is worth continuing trading utilizing this strategy. After the trader has made more trades it would be a good idea to recalculate the average return and make the final conclusion based on more statistical data.

We should also note that this complication is related exclusively to small statistics. If a trader makes 50 and more trades he must take into account all trades without any special considerations.

## Using Trading Strategies and Trading Psychology

This short section is very important. We wrote this section after analysis of our own mistakes and we hope a reader will learn from our experience how to avoid some typical mistakes.

Suppose a trader performs a computer analysis and develops a good strategy, which requires holding stocks for 5 days after purchase. The strategy has an excellent historical return and behaves well during bull and bear markets. However, when the trader starts using the strategy he discovers that the average return for real trading is much worse. Should the trader switch to another strategy?

Before making such a decision the trader should analyze why he or she is losing money. Let us consider a typical situation. Consider hypothetical distributions of historical returns and real returns. They are shown in the next figure.


## Hypothetical distributions of the historical and real trading returns

This figure shows a typical trader's mistake. One can see that large positive returns (> $10 \%$ ) are much more probable than large negative returns. However, in real trading the probability of large returns is quite low.

Does this mean that the strategy stops working as soon as a trader starts using it? Usually, this is not true. In of most cases traders do not follow strategy. If they see a profit
$10 \%$ they try to sell stocks or use stop orders to lock in a profit. Usually the stop orders are executed and the trader never has returns more than 10-15\%.

On the other hand if traders see a loss of about $10 \%$ they try to hold a stock longer in hope of a recovery and the loss can become even larger. This is why the distribution of returns shifts to the left side and the average return is much smaller than historical return.

## Conclusion:

If you find a profitable strategy - follow it and constantly analyze your mistakes.

## Trading Period and Annual Return

To calculate the average annual return one needs to use the average daily growth coefficient calculated for the whole trading capital. Let us remind the reader that this coefficient should be calculated as an average ratio
$K i=\langle C(i) / C(i-1)\rangle$
where $\mathbf{C}(\mathbf{i})$ is the value of the capital at the end of $\mathbf{i}$-th trading day. The average growth coefficient must be calculated as the geometric average, i.e. for $\mathbf{n}$ trading days

| Kav - the average daily growth coefficient |
| :--- |
| Kav $=\left(K_{1} * K 2 * \ldots * K n\right) \wedge(1 / n)$ |

How should one calculate the average daily growth coefficient if a trader holds the stocks for 2 days, spending the entire capital to buy stocks? This method of trading can be presented graphically as

| BUY | HOLD | SELL BUY | HOLD | SELL <br> BUY |
| :---: | :---: | :---: | :---: | :---: |

Suppose that the average growth coefficient per trade (not per day!) is equal to $\mathbf{k}$. This can be interpreted as the average growth coefficient per two days. In this case the average growth coefficient per day Kav can be calculated as the square root of $\mathbf{k}$

## $K a v=k^{\wedge}(1 / 2)$

The number of days stocks are held we will call the trading period. If a trader holds stocks for $\mathbf{N}$ days then the average return per day can be written as

> | Kav - the average daily growth coefficient |
| :--- |
| $\mathbf{k}$ - the average growth coefficient per trade |
| $\mathbf{N}-$ holding (trading) period |
| Kav $=k^{\wedge}(1 / N)$ |

The average annual capital growth Kannual (the ratio of capital at the end of the year to the initial capital) can be calculated as

## Kannual $=k^{\wedge}$ (250/N)

We supposed that the number of trading days per year is equal to 250 . One can see that annual return is larger for a larger value of $\mathbf{k}$ and it is smaller for a larger number of $\mathbf{N}$. In other
words, for a given value of return per trade the annual return will suffer if the stock holding period is large.

Which is better: holding stocks for a shorter period of time to have more trades per year or holding stocks for a longer time to have a larger return per trade $\mathbf{k}$ ?

The next graph illustrates the dependence of the annual growth coefficient on $\mathbf{k}$ and $\mathbf{N}$.


The annual capital growth $K($ annual ) as a function of the average growth coefficient per trade $\mathbf{k}$ for various stock holding periods $\mathbf{N}$

Using this graph one can conclude that to have an annual capital growth equal to about 10 (900\% annual return) one should use any of following strategies:
$N=1$ and $k=1.01$
$N=2$ and $k=1.02$
$N=3$ and $k=1.03$
In the first case one should trade stocks every day, which substantially increases the total brokerage commissions. Let us consider this important problem in detail.

Suppose that the round trip brokerage commissions and bid-ask spread equal (on average) $1.5 \%$ of the capital used to buy a stock. In this case the first strategy becomes unprofitable and profits from other two strategies are sharply reduced. One needs to have much larger profit per trade to have a large annual return. For annual capital growth about 10 the strategies with $\mathbf{N}=1,2,3$ should have growth coefficients per trade $\mathbf{k}$ as large as
$N=1$ and $k=1.025$
$N=2$ and $k=1.035$
$N=3$ and $k=1.045$
If a trader uses a strategy with a trading period of two and more days he can divide his trading capital to buy stocks every day. In this case the risk of trading will be much lower. This important problem will be considered in the next section.

## Theory of Diversification

Suppose that a trader uses a strategy with the holding period $N=2$. He buys stocks and sells them on the day after tomorrow. For this strategy there is an opportunity to divide the trading capital in half and buy stocks every day, as shown in the next table

| First half of <br> capital | BUY | HOLD | SELL | BUY | HOLD | SELL <br> BUY | HOLD | SELL <br> BUY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Second half <br> of capital |  | BUY | HOLD | SELL | BUY | HOLD | SELL <br> BUY | HOLD |

Every half of the capital will have the average annual growth coefficient

## Kannual (1/2) $=\mathbf{k}^{\text {^(250/2) }}$

and it is easily to calculate the annual growth coefficient (annual capital growth) for the entire capital Kannual.

## Kannual = (Capital after 1 year) / (Initial Capital)

Let CAP (0) denotes the initial capital and CAP (250) the capital after 1 year trading. One can write

```
Kannual = CAP (250)/ CAP (0) = Kannual (1/2) = k^ (250/2)
```

Correspondingly for $\mathbf{N}=3$ one can write

```
Kannual = CAP (250)/ CAP (0) = k ^ (250/3)
```

and so on. One can see that the formula for annual capital growth does not depend on capital division. The only difference is the larger influence of brokerage commissions.

However, if we consider the risk of trading when the capital is divided we can conclude that this method of trading has a great advantage!

To calculate the risk for the strategy with $\mathbf{N}=2$ (as an example) one can use an equation

where SQRT is a notation for the square root function and $s$ is the risk of a trade. It was supposed that the trader buys one stock per day. For any N one can obtain

## $\mathbf{S}=\mathbf{s} / \mathbf{S Q R T}(\mathbf{N})$

The larger $\mathbf{N}$ is, the smaller the risk of trading. This is related to dividing capital diversification. However, the more you divide your capital, the more you need to pay commissions.

Mathematically, this problem is identical to the problem of buying more stocks every day. The risk will be smaller, but the trader has to pay more commissions and the total return can be smaller. What is the optimal capital division for obtaining the minimal risk to return ratio? Let us consider an example, which can help to understand how to investigate this problem.

## Problem:

Suppose, a trader has $\$ 5,000$ to buy stocks and he does not use margin. The brokerage round trip commissions are equal to $\$ 20$. The average return per trade (after taking into account the bid-ask spread) is equal to $\mathrm{R}(\%)$. The returns have distribution with the standard deviation (risk) s . What is the optimal number of stocks N a trader should buy to minimize the risk to return ratio?

## Solution:

To buy one stock a trader can spend ( $5000 / \mathbf{N}-10$ ) dollars. The average return $\mathbf{R 1}$ per one stock will be equal to
$R 1=100 \% *[(5000 / N-10) R / 100-10] /(5000 / N)=100 \% *[(5000-10 N) R / 100-10 N] / 5000$
This is also equal to the average return Rav of the entire capital because we consider return in \%.

## $\mathbf{R a v}=\mathbf{R 1}$

One can see that increasing $\mathbf{N}$ reduces the average return and this reduction is larger for a larger value of brokerage commissions. The total risk $\mathbf{S}$ is decreased by $\mathbf{1 / S Q R T}$ ( $\mathbf{N}$ ) as we discussed previously.

## $\mathbf{S}=\mathbf{s} / \mathbf{S Q R T}(\mathbf{N})$

The risk to return ratio can be written as

```
S/R \(=\) 5000*s/ \{SQRT (N)* 100\% * [(5000-10N) R/ 100-10N]\}
```

The problem is reduced to the problem of finding the minimum of the function $\mathbf{S} / \mathbf{R}$. The value of $s$ does not shift the position of the maximum and for simplicity we can take $s=1$. The function $\mathbf{S} / \mathbf{R}$ is not simple and we will plot the dependence of the ratio $\mathbf{S} / \mathbf{R}$ for different values of $\mathbf{N}$ and $\mathbf{R}$.


The risk to return ratio as a function of number of stocks

One can see that for the average return per trade $\mathbf{R}=1 \%$ and for the $\$ 5000$ capital the optimal number of stocks when the risk to return ratio is minimal is equal to 1.5 . Therefore, a trader should buy 1 stock one day, 2 stocks another day, ... For $\mathbf{R}=1.5 \%$ the optimal number of stocks is equal to 2.5. If $\mathbf{R}=2 \%$ then $\mathbf{N}=3.5$. The problem is solved.

Let us consider the "value of payment" for lower risk to return ratio.

## Case 1. $R=1 \%$

If a trader buys one stock he or she would pay $\$ 20$ in commissions and the average profit is equal to $(5000-10) * 0.01-10=\$ 39.9$

In this case the optimal number of stocks to buy is equal to $N=1.5$. The average round trip commissions are equal to $\$ 20 * 1.5=\$ 30$

The average profit can be calculated as
$(5000-15) * 0.01-15=\$ 34.85$
Therefore, a trader is losing about $\$ 5$ per day when he buys 1.5 stocks. This is equal to $13 \%$ of the profit. This is the payment for lower risk.

## Case 2. R = 1.5\%

If a trader buys one stock he or she would pay $\$ 20$ in commissions and the average profit is equal to $(5000-10) * 0.015-10=\$ 64.85$

In this case the optimal number of stocks to buy is equal to $N=2.5$. The average round trip commissions are equal to $\$ 20 * 2.5=\$ 50$

The average profit can be calculated as
$(5000-25) * 0.015-25=\$ 49.63$
Therefore, the trader pays about $\$ 15$ for lower risk. This is equal to $23 \%$ of the profit.

## Case 3. $\mathrm{R}=2 \%$

If a trader buys one stock the average profit is equal to
$(5000-10) * 0.02-10=\$ 89.8$
In this case $N=3.5$. The average round trip commissions are equal to
$\$ 20$ * 3.5 = \$70
The average profit is equal to
$(5000-35) * 0.02-35=\$ 64.3$
Therefore, the trader pays about $\$ 25$ for lower risk. This is equal to $28 \%$ of the profit.
You can ask why we should lose so much in profit to have lower risk? The answer is simple. This is the only way to survive in the market with a small initial trading capital. After a couple of months of successful trading your capital will be larger and the influence of commissions will be much smaller.

## PART 3

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1. Random walk and stop-limit strategy
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3. How to make a profit in the non-random market
4. What can be wrong
5. Stops and real trading

## Random walk and stop-limit strategy

One of the most popular stock market theories is the random walk model. It is assumed that for short periods of time stock prices change randomly. So, probabilities of growth and decline are equal. One of the most important theorems of the random walk model can be formulated for the stock market as:

For any initial stock price and for any limit price the probability of hitting the limit price is equal to 1 .

From the first point of view making profit is very easy: buy stock, place the limit order to sell above the stock price and wait. Sooner or later the limit will be touched and you can make a profit.

This is wrong! There is no chance of making any profit in the random market. You should remember that the stock price can also touch the level $=0$ (or very low price) and the game is over. To show how to analyze the trading strategies in the random market consider stop-limit strategy. The stop level can be equal to zero, which corresponds to a game without any stop.


On the diagram, $S$ and $L$ are the differences between the current stock price and the stop and limit order levels. The simplest strategy is to buy some stock and wait until the stock price touches the stop or limit levels (prices). If the stop level is touched first - you are a loser.

If the stock price touches the limit level - you sell the stock with a profit and you are a winner.

It can be shown that for this model the probability of touching the stop level can be written as
$P(S)=L /(S+L)$
One can find the derivation of this equation in William Feller's book Introduction to Probability Theory and Its Application. The probability of touching the limit level is equal to
$P(L)=S /(S+L)$
One can check that $P(L)=1-P(S)$. The average return per trade can be written as

$$
R=L * P(L)-S^{*} P(S)=0
$$

This strategy gives zero average return in the case of zero commissions. It can be shown that the variance of the returns (the squared standard deviation or "squared risk") is equal to
$\mathbf{s}^{\mathbf{2}}=\mathbf{S *} \mathbf{L}$
So, the larger the deviations of the stop or limit order levels from the current stock price, the larger the risk of this trading strategy.

You can consider many other trading strategies but it can be shown that for random walk price changes the average returns are always equal to zero for zero transaction costs. In real life you will be a loser because of brokerage commissions and bid-ask spreads.

## Non-random walk and stop-limit strategy

The situation changes when the probability of growth is larger than the probability of decline. In this case the probability of touching the limit level can be higher than the probability of touching the stop level and your average return is positive.

Consider a simple case. Suppose that the stock price is equal to $\$ 100$ and the price is changing by one dollar steps. As in the previous section let $\mathbf{L}$ and $\mathbf{S}$ be the deviations of the limit and stop orders from the stock price. Denote by $\mathbf{p}$ the probability of stock price gain and by $\mathbf{q}$ the probability of stock price decline. The sum of these probabilities is equal to 1.
$\mathbf{p + q}=\mathbf{1}$
The probability of touching the stop level can be written as
$P(S)=\left[1-(p / q)^{\wedge} L\right] /\left[1-(p / q)^{\wedge}(S+L)\right]$
The probability of touching the limit level can be calculated from
$\mathbf{P}(\mathrm{L})=\mathbf{1 - P ( S )}$
The average return can be calculated from the equation

## $\mathbf{R}=\mathbf{L} * \mathbf{P}(\mathbf{L}) \mathbf{- S * P ( S )}$

What are the optimal limit and stop orders in the case when $p$ is not equal to $q$ ? The next figure shows the results of calculation of the average returns per trade for various values of the stop and limit orders.


Average return per trade for stop-limit strategy for two values of the growth probability $\mathbf{p} . \mathrm{S}$ is the deviation of the stop level from the current stock price in \%

From this figure one can draw two very important conclusions:

- if a trader selects stocks with a growth probability larger than $50 \%$ then the average return is positive and the stop levels must be far away from the current stock price to obtain the maximal return.
- if a trader select stocks with a growth probability smaller than $50 \%$ then the average return is negative and the stop levels must be as close as possible to the current stock price to minimize losses.

There is an important theorem for the non-random walk model: if the limit level is equal to infinity (no limit) then the probability of touching the stop level is equal to:
$P=1$
if $p<=q$
$P=(q / p)^{\wedge} S$ if $p>q$
Therefore, if $\mathbf{p}>\mathbf{q}$ (bullish stock) there is a chance that the stop level will never be touched. This probability is larger for larger $\mathbf{S}$ and smaller ratio $\mathbf{q} / \mathbf{p}$.

## How to make a profit in the non-random market

It is hard to imagine that all stocks have a probability of growth exactly equal to $50 \%$. Many stocks have a growth probability of $55 \%, 60 \%$ and more. However, the stock market does not grow with a high probability and this means there many stocks with low growth probabilities $45 \%, 40 \%$ and less. The probabilities depend on technically overbought or oversold conditions, good or bad fundamentals, interest from traders, etc. How can we apply stop-limit strategy for this market to obtain a profit?

To answer this question, consider a simple case. Suppose that the "market" consists of two stocks. One stock has the growth probability $\mathbf{p 1}=0.55$ and the second stock has the growth probability $\mathbf{p 2} \mathbf{= 0 . 4 5}$. We buy the two stocks using equal amounts of money for each stock. We also place stop loss orders and place mental limit orders to sell if we see some profit.

Denote by $\mathbf{S}$ and $\mathbf{L}$ the differences between the current stock price and the stop and limit order levels. The next figure shows the average returns per trade for this strategy depending on the levels of stop and limit orders.


## The average return as a function of the levels of limit order for various stop loss order levels. Details of this trading strategy are described in the text

It is important to note that using stop loss orders which are far away from the current stock price together with close limit orders provide negative returns. The winning strategy is using "tight" stops and large limit targets.

## What can be wrong

From the previous section one can conclude that making money in the stock market is quite easy. All you need is to have enough money to buy a dozen of stocks, place stops and limits and wait for a nice profit. However, the majority of traders are losing money. What is wrong?

In the previous section we supposed that stocks have growth probabilities, which are stable. So, if $\mathbf{p}=0.55$ at the moment of purchase it will be equal to 0.55 even after sharp price growth. This is wrong. One can find a stock with $\mathbf{p}=0.55$ and even larger but after a couple of days this probability usually becomes close to 0.5 or even becomes less then 0.5 , which is the reason for price fluctuations.

The waiting time when your limit order will be executed can be very long. Many times you will sell stocks early with smaller profit.
If you select stocks randomly, you will mostly select stocks with growth probabilities close to $50 \%$. If we consider the model described in the previous section with $\mathbf{p 1}=0.49$ and $\mathbf{p 2}=0.51$ then this scheme will not work. Brokerage commissions and bid-ask spreads will eat up your profit.

What can be done? You need to select stocks with high probabilities of growth and optimize the stop order levels. In the next section we present the results of computer analysis of the real market to show methods for developing a winning trading strategy.

## Stops and real trading

Suppose you randomly select a stock, buy it and want to place a stop loss order to prevent your trading capital from a large loss. What is the probability of touching your stop order level? How does it depend on the stock volatility? How will the probability of touching stop change in two days? Five days?

To answer these questions we have performed a computer analysis of the 11 years history of 300 randomly selected actively traded stocks.

The stop order will be executed during a designated time period if the minimal stock price for this period is lower than your stop order level. Suppose you bought a stock at the market closing at the price CLO. Denote by MIN the minimal stock price during a certain period of time. We will study the distribution of the difference
$x=$ MI N - CLO
which characterizes the maximum of your possible loss on the trade for the considered period of time. The next figure illustrates this statement.


## The difference MI N - CLO characterizes the maximal loss from the trade during 11 trading days after the stock purchase

The average value of MIN - CLO depends on the stock's volatility. There are many ways of defining the volatility. We will consider the average amplitudes of daily price change
$A=<M A X-M I N / 2$
where the averaging has been performed for a one-month period. It is obvious that the average value of <MIN - CLO> must depend on the amplitude A. We have shown that <MIN - CLO> is approximately equal to negative value of $\mathbf{A}$, i.e.
$<\mathrm{MIN}-\mathrm{CLO}>=-\mathrm{A}$
where MIN is the minimal stock price during the next day after purchase. The correlation coefficient of this linear dependence is equal to 0.5 . For a longer period of time the value of <MIN - CLO> can be much less than -A.

You cannot expect that placing a stop order at a level, which is slightly less than -A will be safe enough to avoid selling the stock due to daily stock price fluctuations. The distribution of $\mathbf{x}=\mathbf{M I N} \mathbf{~ - ~ C L O ~ i s ~ r a t h e r ~ b r o a d . ~ T h e ~ n e x t ~ f i g u r e ~ s h o w s ~ t h e s e ~ d i s t r i b u t i o n s ~ f o r ~ o n e ~ a n d ~ t e n ~}$ days after stock purchase.


## Distribution of (MI N - CLO) / A for randomly selected stocks

One can see that the distributions of MI N - CLO are rather broad and non-symmetrical. There is a little chance that the minimal stock price will always be higher than the purchase price CLO. The distribution become broader for longer stock holding periods. The average (mean of the distribution) becomes more and more negative. The next figure shows the time dependence of the average values $<(\mathbf{M I N} \mathbf{N} \mathbf{C L O}) / \mathbf{A}>$ and the standard deviations of the distribution of this value.


Time dependence of the average values <(MIN-CLO) / A> and the standard deviations of the distribution of this ratio

One can see a strong increase in the width of distribution with time.
The crucial question is the probability of touching the stop levels. We performed the calculations of these probabilities for various stop levels. The next figure presents the results.


Probabilities of touching stop levels during 1, 2, 4 and 8 days for randomly selected
stocks. A is the daily price amplitude defined in the text.
One can see that if the stop loss order is placed at -A level then the probability of execution of the stop order during the next day after the purchase is equal to $45 \%$. The probability becomes equal to $60 \%$ if you hold the stock for two days.

If you hold the stock for 8 days then the probability of executing the stop order becomes $80 \%$ ! If you place the stop at the -3A level than the probability of stop order execution remains less than $50 \%$ even if you hold the stock for 8 days.

Using this plot you can calculate the average loss per trade. Let us offer you an example.

Suppose you bought a stock at $\$ 100$. The daily price amplitude is equal to $\$ 3$. Suppose you hold the stock for 4 days. To calculate the average loss due to execution of the stop order you need to multiply the difference CLO - STOP by the probability of touching the stop level.

## AVERAGE LOSS $=($ CLO - STOP) $\boldsymbol{*}$ PROBABILITY

Here is a table that can help you to select the optimal stop loss level.

| STOP | CLO - STOP | PROBABILITY |
| :---: | :---: | :---: |
| 97 | 3 | 0.71 |
| AVERAGE LOSS |  |  |
| 94 | 6 | 0.49 |
| 2.13 |  |  |
| 91 | 9 | 0.32 |
| 2.94 |  |  |
| 88 | 12 | 0.19 |
| 2.88 |  |  |
| 85 | 15 | 0.11 |

It is interesting that the worst decision you can make for this trade is placing the stop at the $\$ 94$ level $(-6 \%)$. Your average loss is maximal at this point!

For practical purposes we publish the figure, which shows dependence of average loss on the stop order level in A units. 1, 2, 4, and 8 days of holding stock are considered.


Dependence of the average loss on the stop order level in $A$ units for 1, 2, 4, and 8 days stock holding

From this figure one can conclude that to minimize the average loss from the stop orders you need to place stops either very close or very far away to the current price. Your decision should be based on the number of days you are going to hold the stock.

One again: all calculations have been made for randomly selected stocks. For these stocks the average growth probabilities are close to $50 \%$. What will happen if we consider specifically selected stocks when the growth probability is not equal to $50 \%$ ?

Before starting to look at this important question we need to consider some technical parameters, which are very helpful in stock selection. They allow to select stocks with the highest growth probabilities and develop very profitable trading strategies. These parameters have been introduced in our book Short-Term Trading Analysis. (See link to Text Level-2 on the page http://www.stta-consulting.com). In the next sections we repeat these definitions.

## PART 4

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## Stock price trends

Trend is a simple intuitive term. If a stock price is increasing one can say the trend of this stock is positive. If a stock is declining - the trend is negative. You can also say: a stock has momentum. This is standard terminology. In many books on technical analysis of the stock market you can read about momentum investments: buy stocks with the highest growth rates.

We need to define trends mathematically. The simplest method is using the linear fit of stock price-time dependence.


Consider $\mathbf{N}$ trading days and draw the fitting line $\mathbf{L}(\mathbf{i})$ through the points $\mathbf{P}_{\mathbf{i}}$, where $\mathbf{P}_{\mathbf{i}}$ is the closing stock price on the $\mathbf{i}$-th day.


The $\mathbf{N}$-th day is the day of interest, the day of the analysis.
$L(i)=A+B i \quad i=1,2, \ldots, N$
The coefficient B is the slope of the fitting line. The slope can be considered as the price trend of the stock. It characterizes the average daily price change (in dollars) during one day. However, this value is not perfect when comparing trends of different stocks.

We will define a trend as the average price change in \% during one trading day. Mathematically this can be written as

Trend $=\mathbf{T}=\langle\Delta \mathbf{P} / \mathbf{P}\rangle * 100 \%$
This definition has one disadvantage. What price $\mathbf{P}$ should be used in this equation? Using the current stock price is not a good idea. In this case, trend will be very volatile. It is better to use a more stable price characteristic: the value of the linear fitting line on day $\mathbf{\# N}$ the day of the analysis
$\mathbf{P}=\mathbf{A}+\mathbf{B}^{*} \mathbf{N}$
The final equation for trend looks like
$T=B /(A+B * N) * 100 \%$
This equation we used in our computer analysis of the stock market. Such a definition of trend substantially reduces the influence of price fluctuations during the final days. The next figure shows an example of trend calculations for a 16 days time frame.


Example of stock trend calculations for a 16 days time frame
Another way to define the trend is by using logarithmic scaling. This is a good idea if you study a stock's long history of price change, for example, from 1 to 100 dollars. For short-term trading this change is unrealistic and we will keep things as simple as possible. So, we will use simple linear fits, and trends are related to the slope of the fitting lines.

## Deviation parameters (D-parameters)

The deviation or D-parameter, which will be defined here, is crucial for short-term stock trading. Briefly, this is a characteristic of deviation of the current stock price from the fitting line. This characteristic is important for the definition of oversold or overbought stocks.

A simplest idea of the definition of overbought and oversold stocks is the location of the stock closing price relative to the trading range. We assume that when the stock price is in the trading range, the stock performance is normal. If the price is out of this range, the stock may be oversold or overbought.


We define trading range as a channel between support and resistance lines. Usually, these lines are determined intuitively from the stock price charts. This is not a good way if you use the computer to analyze stock price performance. The line drawing depends on the trader's skill and this method can be used only for representation of stock performance in the past. Some traders use minimal and maximal prices to draw the support and resistance lines, others like closing prices. There is a problem of what to do with points, which are far away from the trading channel, etc.

We suggest using the standard deviation $\sigma\left(\sigma^{2}=\left\langle\left(\mathbf{P}_{\mathbf{i}}-\mathbf{A}-\mathbf{B i}\right)^{\wedge} \mathbf{2}>\right)\right.$ for the definition of the support and resistance lines. Mathematically this can be written as

```
Support line = A + Bi- \sigma
Resistance line = A + Bi + \sigma
```

where $\mathbf{A}+\mathbf{B i}$ is the equation of the linear fitting stock price time dependence; i is the day number.

One should notice that the definition of the support and resistance lines depends on the number of trading days being considered. You should always indicate what time frame has been used.

To characterize the deviation of the stock closing price from the fitting line we have introduced a new stock price characteristic: deviation, or D-parameter.
$D=\left(P_{N}-A-B N\right) / \sigma$
where $\mathbf{N}$ is the number of days which were used for linear fitting, $\mathbf{P}_{\mathbf{N}}$ is the closing stock price on day $\# \mathbf{N}$ (the day of the analysis), $\mathbf{A}$ and $\mathbf{B}$ are the linear fitting parameters, and $\sigma$ is the standard deviation of closing prices from the fitting line.

Let us explain the meaning of this equation. The difference ( $\mathbf{P}_{\mathbf{N}}-\mathbf{A}-\mathbf{B N}$ ) is the deviation of the current stock price from the fitting line. D-parameter shows the value of this deviation in $\sigma$ units.

## D $<-1$ the stock may be oversold

D > +1 the stock may be overbought
One can say the words "oversold" and "overbought" should be used together with the number of trading days considered. So it more precise to say: the stock is oversold in the 30 days time frame.

Here we have to note that comparison $\mathbf{D}$ with +1 or -1 is not optimal and depends on the time frame. In the next section we will consider an approach, which allows us to calculate the values of the $\mathbf{D}$ and $\mathbf{T}$ parameters so as to optimize the selection of oversold and overbought stocks.

## Returns of overbought and oversold stocks

To check the hypothesis about our criteria of overbought and oversold conditions one needs to calculate the average returns for stocks for which the $\mathbf{D}$ parameter is negative or positive. Here we have to note that oversold and overbought conditions become more pronounced if one also considers the trends $\mathbf{T}$. It is obvious (and this has been checked by computer analysis) that if the trend is negative and $\mathbf{D}$ is also negative the oversold condition becomes stronger.

How can we determine the range of the $\mathbf{D}$ and $\mathbf{T}$ parameters for oversold stocks? Let us start with an analysis of distributions of these parameters. 16 days stock price history will be considered in this section, i.e. for calculation of $\mathbf{D}$ and $\mathbf{T}$ one needs to download stocks prices for the last 16 trading days. On the next figure the distributions of $\mathbf{T}$ (16) and $\mathbf{D}$ (16) are shown. Later we will drop the (16) notation, but one needs to remember that for other historical time frames the distributions have substantially different shapes.


Distribution of the deviations ( $D$ ) and trends ( $T$ ) calculated for a $\mathbf{1 6}$ days stock price history.
From the first point of view we need to consider stocks with very low values of $\mathbf{T}$ and $\mathbf{D}$ to be sure that these stocks are oversold. Yes, theoretically this is true. Computer analysis shows that the smaller $\mathbf{T}$ and $\mathbf{D}$, the larger the probability of positive returns. Returns of stocks with $\mathbf{T}<-1 \%$ and $\mathbf{D}<-1.5$ are very large.

In practice, selecting stocks with such extreme values of deviations and trends yields smaller annual returns. The number of these stocks is small a trader is able to find such stocks only one or two times a month. As we analyzed in the previous sections, the annual return will be very small.

It is more effective to select stocks with softer conditions but the probability of finding such stocks is higher. We suggest defining oversold stocks as stocks for which

## T < Tav - (Standard Deviation of T distribution) <br> D < Dav - (Standard Deviation of D distribution)

Correspondingly the overbought stocks are the stocks for which

## T > Tav + (Standard Deviation of T distribution) <br> D > Dav + (Standard Deviation of D distribution)

We will study the average returns

```
Return = [CLO (N) - CLO] / CLO * 100%
```

where CLO is the closing stock price on the day of the analysis and CLO(N) is the closing stock price on day $\# N$ after the day of the analysis. The next scheme illustrates our definitions.


Here, a denotes the day of the analysis $(\mathbf{N}=0)$. The distributions of the $\mathbf{D}$ and $\mathbf{T}$ parameters are shown on the next figure.

The standard deviation of distribution of the D-parameters is equal to 1.185. For the $\mathbf{T}$ parameter the standard deviation is equal to 0.558 . Therefore, we will define oversold and overbought stocks within 16 days frame as

## Oversold stocks: $T<-0.5581$ and $D<-1.185$ <br> Overbought stocks: T > 0.5581 and D > 1.185

The next figure shows the average returns of the oversold and overbought stocks as a function of number N of days following the day of the analysis. For comparison we present the average returns of randomly selected stocks


The average returns (CLO (N) - CLO) / CLO * 100\% of the oversold and overbought stocks as a function of number $\mathbf{N}$ of days following the day of the analysis. The average returns of randomly selected stocks (black squares) are presented for comparison.

Let us formulate some conclusions from this plot.

- The average return of randomly selected stocks is a linear function of time with a positive slope. This is related to the bull market of the 90's, when the stock price history was studied. - The average returns of the oversold stocks is much larger than the average returns of the overbought stocks. This effect is more pronounced for short periods of stock holding.
- If a trader buys oversold stocks or sells short overbought stocks he(she) should not hold these stocks for a long time. It is better to close position in three to five days and switch to other stocks with higher potential short-term returns.


## Optimal stops for oversold stocks

Now we are ready to consider an analysis of optimal stop levels for specifically selected stocks. As an example we will consider oversold and overbought stocks within 16 day time frame. Let us study the correlation of deviation (D) and trend (T) parameters with the minimal stock price during N following days after the day of analysis.
The next figure presents the average values of (MIN - CLO) / A in the same way as we did previously. Here, $\mathbf{A}$ is the average daily stock price change as we defined before. The open squares show results for randomly selected stocks to compare these data with our previous analysis.


The average values of (MIN-CLO) / A for oversold and overbought stocks within 16 days time frame. Open squares show results for randomly selected stocks

It is interesting to note that for oversold stocks the minimal prices for the first four days after the day of the analysis are very close to the minimal prices of randomly selected stocks. This is despite the large positive move of the closing price of these stocks.

This phenomenon is related to the higher volatility of oversold stocks during the first days of trading after large drops in the stock price. Many people get upset over these stocks and continue selling. On the other hand, bottom fishers buy these stocks, and in the end the bulls win this game.

A trader should be prepared to overcome the difficulty of observing drops in stock price during the competition between bulls and bears. One needs to have patience and wait for a positive price move to sell the stock with a profit. Statistically such an approach is a winning game, but one should remember that statistics always assume a distribution of return and possible losses are likely.

Let us consider very important question: how can we place an optimal stop loss order to minimize losses and to obtain a good average return? The next figure presents the results of calculation of the average returns (they were defined previously) as a function of number of stock holding days at various levels of stops. The parameter $S$ is defined as


## Average returns as a function of number of stock holding days at various levels of stops. Oversold stocks are considered.

From this figure one can conclude that for oversold stocks using any stops decreases the average return. The worst thing what a trader can do is place stops close to the - $\mathbf{A}$ level. If a trader holds stocks for a period less than 4 days it is worth considering stops, which are placed very close to the CLO price. The change of the average return is not so significant. The next figure presents the average returns as a function of levels of stops for a four days stock holding period.


## Average returns as a function of levels of stops for a four days stock holding period

One can see that the best returns are obtained when a trader uses stops, which are located very far away from the purchase price or trades stocks without any stops.

This contradicts the popular opinion that profitable trading without stops is impossible. How one can handle trading without stops? It is possible only in one case: the trading capital
must be large enough to be able to buy many stocks. In this case, even a $50 \%$ price drop of one or two stocks will not kill a trader. Otherwise, trading without stops is financial suicide.

For a small capital - one which allows holding two to four stocks in the portfolio it is better to use stops and place them either very close to the purchase price or lower than -5A level. The average return will be less, but the trader will survive.

Let us repeat once again: trading stocks without stops is possible only for experienced traders, who are very sure about their stock selection. They must use well-tested strategy and buy many stocks so as to minimize the risk of a sharp drop in their trading capital.

What happens if a trader make mistakes and buys stock with very low growth potential? This can happen due to a bear market, choosing a wrong industry, or just bad luck. This problem will be considered in the next section.

## Stop strategy for inexperienced traders

Let us suppose that a trader is a novice in the stock market. He is trying to beat the market by using a trading strategy, which seems to be profitable because it allows him to trade like many other traders. This is a typical mistake. The majority of traders are losing money and any strategy, which is similar to other people's strategies, will not be very profitable.

The problem is how to survive in the stock market game while testing a new strategy?
Suppose that our novice uses popular momentum strategy: he buys stocks with the highest price rise during the last days of the rise. We can simulate such a strategy by consideration of overbought stocks. As in the previous sections we will consider a 16 days history period for calculation of the D and T parameters.

The simplest way to survive in the stock market is by using stop loss orders. The next figure shows average returns as a function of number of stock holding days at various levels of stops. The no-stop-loss-orders strategy is shown for comparison.


Average returns as a function of number of stock holding days at various levels of stops. Overbought stocks are considered

Let us list the conclusions that can be drawn from analysis of this figure.

- For very short periods of stock holding (one to four days) using stops does not earn returns worse than the returns from the no-stop-loss-orders strategy.
- The best results are obtained using very tight stops.
- For longer periods of stock holding closer stops (about 1-2 A) earn returns worse than those of the no-stop-loss-orders strategy.
- If you do not like tight stops and you are going to hold stocks for a long period of time then it is better to use stops which are lower than -5A.

Let us suppose that our novice has got some experience, has understood that his strategy was bad, and now his understanding allows him to choose stocks with about $50 \%$ probability of growth. This case will be considered in the next section.

## Stop strategy for an average trader

The case, which we are going to consider, is close to the stop-limit strategy for a mixture of stocks with both high and low probabilities of growth. This problem was theoretically considered in one of the previous sections. How can we apply this strategy in practice?

It is hard to place simultaneously the limit and stop orders for one stock. So, we will consider a strategy, which assumes selling stocks after N days of holding. The problem is the determination of the optimal stops to cut stocks with low probabilities of growth. Consider the average returns of the strategy as a function of stop levels. We will assume that $50 \%$ of selected stocks are oversold and $50 \%$ of the stocks are overbought. This is a good model for the average stock selection.


Average returns as a function of number of stock holding days at various levels of stops. A mixture of oversold and overbought stocks is considered

The figure is very similar to the figure in the previous section. The largest return can be obtained if the trader uses very tight stops. The results in this case will be much better than the return without using any stops. Stop orders that are far away from the purchase price produce better returns than stops around the - $\mathbf{A}$ level.

## Stock volatility

All previous results have been presented via the daily stock price amplitudes $\mathbf{A}$, which can be a characteristic of stock volatility. You probably know many other definitions of this parameter. In this section we will consider methods of calculations of stock volatility and will show that stock volatility is a function of the $T$ and $D$ parameters.

There are hundreds methods of defining the stock volatility. The amplitude A is measured in dollars, and it is better to introduce a new parameter to compare one stock to another. Let us define the stock volatility in two different ways

```
V1 (t) = (MAX (t) - MIN (t))/(MAX (t) + MIN (t)) * 100%
V2(t) = < (MAX (t) - MIN (t-1))/(MAX (t) + MIN (t)) * 100% >
```

where $\mathbf{V 1}(\mathbf{t})$ and $\mathbf{V 2 ( t )}$ are stock volatilities referring to day $t$. V1(t) describes relative volatility during the day $\mathbf{t}$ and $\mathbf{V 2 ( t )}$ describes relative volatility during days $\mathbf{t}$ and $\mathbf{t}$-1. After averaging one can write

```
V1 = <V1 (t)>
V2 = <V2 (t)>
```

where the angular brackets <...> denote the averaging over some period of time. In this work we perform one month averaging. The first parameter V1 (t) can be also written via the daily amplitude $\mathbf{A}(\mathbf{t})$.

V1 (t) $=A(t) / P(t) * 100 \%$
where $\mathbf{P}(\mathbf{t})$ is the average price during trading day $\mathbf{t}$
$P(t)=(\operatorname{MAX}(t)+M I N(t)) / 2$
The next figure illustrates these definitions.


Day \#
I llustration for the definitions of stock volatilities
The values of V1 and V2 are very close to each other. The next figure shows V1 and V2 for 250 stocks.


One can conclude that the daily amplitude $\mathbf{A}$, or the relative volatility $\mathbf{V 1}$ which is related to this value, is a good characteristic of the short-term stock volatility. There many other definitions of stock volatilities related to the closing prices only. However, for our purposes, for which we need to study stop orders, it is necessary to consider minimal and maximal daily prices.

To give you some idea about the values of the daily amplitudes we will show the values of V1 for some active stocks. The next table shows stocks with large V1 values.

| Ticker | V1, \% |
| :---: | :---: |
| VTSS | 3.27 |
| ASND | 2.67 |
| QCOM | 2.48 |
| DIGI | 2.47 |
| KLAC | 2.46 |
| LSI | 2.46 |
| XLNX | 2.43 |
| AOL | 2.43 |
| PSFT | 2.37 |
| MU | 2.33 |
| DELL | 2.32 |
| ENTM | 2.30 |
| PMTC | 2.27 |
| NXTL | 2.26 |
| AMAT | 2.23 |
| ORCL | 2.20 |
| COMS | 2.18 |
| CHRS | 2.18 |


| BGEN | 2.08 |
| :---: | :---: |
| BBBY | 2.08 |
| AMD | 2.07 |
| SUNW | 2.05 |
| INGR | 2.04 |
| LLTC | 2.03 |
| NOVL | 2.02 |

The next table shows stocks with small V1 values.

| Ticker | V1, \% |
| :---: | :---: |
| DOW | 0.88 |
| SBC | 0.88 |
| AIG | 0.88 |
| CHV | 0.85 |
| DOV | 0.84 |
| AIT | 0.83 |
| GIS | 0.82 |
| AN | 0.82 |
| XOM | 0.82 |
| GSX | 0.80 |
| MHP | 0.80 |
| CLX | 0.79 |
| TMM | 0.79 |
| SBH | 0.79 |
| GRN | 0.77 |
| MMC | 0.77 |
| VO | 0.76 |
| SPC | 0.76 |
| ED | 0.75 |
| BTI | 0.74 |
| RD | 0.72 |
| BBV | 0.58 |
| BP | 0.57 |
| SC | 0.56 |
| AEG | 0.54 |

To use this table to estimate the daily amplitude A one needs to multiply the V1 value by the current price and divide the result by 100.

However, using average values of V1 can be dangerous if a trader is going to buy oversold or overbought stocks. The volatilities of these stocks are higher. The next figure shows the average values of the relative volatilities V1 calculated for randomly selected stocks and for overbought and oversold stocks.


The average values of the relative volatilities V1 calculated for randomly selected stocks and for overbought and oversold stocks. Vertical bars show the standard deviations of the distributions of V1

One can see than the maximal relative volatilities V1 are observed for oversold stocks.

## Trading strategy using limit orders

Using limit orders to sell is very popular among novices. They buy stocks, place the sell limit order and wait for the stocks to touch this limit. Unfortunately, this is a not a good strategy. There is a non-zero probability of complete disaster. The wait for the limit to be touched can be very long and during this time the stock price can go to very low levels. Using the non-random walk model it can be shown that the limit order will never be executed with probability $\mathbf{P}$

$$
P=(p / q)^{\wedge} L \quad \text { if } \quad q>p
$$

Let us remind the reader that $\mathbf{p}$ is the growth probability and $\mathbf{q}=\mathbf{1 - p}$ is the probability of decline. $L$ is the difference between the limit order level and the current stock price. Details of this model have been considered previously.
Using limits can reduce the average returns per trade even in the case of buying oversold stocks. The next figure shows the results of calculating the dependencies of average returns on the number of stock holding days for different levels of limits $\mathbf{L}$.

L = LIM-CLO


## The dependencies of average returns on the number of stock holding days for different levels of limits

One can see that the worst results are obtained when a trader uses limits, which are very close to the price of purchase.

## Limits, stops and risk

We have finished our short discussion of the influence of stop and limit orders on the average return per trade. However, we can be asked: well, limits and stops reduce the average returns. How about the risk? Maybe it is better to have a smaller return but smaller risk.

We agree with this argument. The only point that needs to be clarified is the relationship between the average return and the risk. We mentioned earlier that the best trading strategy is the strategy with the minimal risk/return ratio. In this section we consider the influence of stops and limits on this ratio.

The trading of oversold stocks will be considered as an example. We calculated the average returns, risk (standard deviation of the return distribution) and the risk/return ratios for various stop and limit values for a four days stock holding period. The next figure shows the result of these calculations for the sell-limit strategy described previously.


## Average returns, risk (standard deviation of the return distribution) and the risk/ return ratios for various limit values for a four days stock holding period. Trading oversold stock has been considered

From this figure one can make a very important conclusion: the risk to return ratio varies very little if the level of the sell limit order is larger than 2.5A. Let us remind the reader that A is the average daily stock price amplitude. Therefore, using limit orders for oversold stocks with high probabilities of growth is not a bad idea.

The risk to return ratios for the strategy with stop loss orders have a more complicated dependence on the stop levels. The next figure shows the results of calculating the average returns, risk, and risk to return ratios for different levels of stop orders. As in the previous case, the strategy of a four days stock holding of oversold stocks has been considered.


## Average returns, risk (standard deviation of the return distribution) and the risk/ return ratios for various stop values for a four days stock holding period. Trading oversold stock has been considered

From the first point of view one can conclude that the best strategy is placing stop loss orders very close to the purchase price. The risk to return ratio of this strategy is the lowest. Theoretically this is correct. However, experienced traders know very well that these stops cannot prevent losses from negative overnight gaps. The opening stock price can be much lower than the closing price of the previous day, and the stop order will be executed at very low level.

The morning flow of sell orders in event of bad news can cause execution of the stop loss order at an almost minimal price during the early selling off. The average return can be much lower than expected. The trader must also remember about bid-ask spreads and brokerage commissions, which also reduce the average return.

It seems that the optimal placing of the stop loss order is lower than $-5 \mathbf{A}$. The risk to return ratio is close to the ratio without stop orders, the average return does not suffer much, and this stop can prevent a big loss of the trading capital.

How much lower? It depends on trader's habits and experience. The average returns and the risk to return ratios do not change much after $-5 \mathbf{A}$. It is much more important to not place a stop loss order in the vicinity of $-2 \mathbf{A}$, where the risk to return ratio is maximal.

We have considered some strategies, which allow us to obtain the maximal average return while minimizing the downward risk. However, if you look closely at the absolute values of returns you can conclude that these returns are small and comparable to the transaction cost (bid-ask spreads and brokerage commissions). How can we increase the absolute values of returns? This question will be considered in the next section.

## I ncreasing average return

There are many ways to improve trading strategies. We have considered optimization of stock holding periods, optimal division of trading capital, and using stops and limits to sell stocks. However, the main source of obtaining better return is a good stock selection. Buying oversold stocks is good strategy, but it can be improved if a trader makes stronger selection. Let us illustrate this idea by the next example.

Selection of oversold stocks within a 16 days time interval allows us to obtain an average return about $2.2 \%$, as described in the section "Returns of overbought and oversold stocks". It is possible to obtain a much better return if one buys stocks on the next day if stock prices decline still further. Denote ( $\mathrm{t}-1$ ) the day of analysis, i.e. the day when the list of
 the market closing, i.e. the purchase price is equal to CLO ( t ). The stocks will be sold on day N at the price CLO $(t+N)$. The next scheme illustrates these notations.


Here, $\mathbf{a}=\mathrm{t}-1$ denotes the day of the analysis, $\mathbf{p}=\mathrm{t}$ denotes the day of the purchase ( $\mathrm{N}=0$ ). The next figure shows the average returns

## $\mathbf{R}=[\mathrm{CLO}(\mathrm{t}+\mathrm{N})-\mathrm{CLO}(\mathrm{t})] / \mathrm{CLO}(\mathrm{t}) * 100 \%$

as a function of $N$. We consider two cases. One is selecting stocks, which dropped during day $t$ (the next day after the day of the analysis) more than $-2 \mathbf{A}$, where $A$ is the daily price amplitude, which was defined previously. The second case is buying stocks, which rose during day $t$ more than $2 \mathbf{A}$.


The average returns of specially selected oversold stocks as a function of the stock holding days

One can see that selecting stocks with large price drops substantially increases the average return and the strategy becomes much more profitable.

If one buys stocks, which started rising, in their price during day $\mathbf{t}$ then the average return is close to zero. This is an example in which momentum strategy (buying rising stocks) is not working.

In this section we have considered improving the average return per trade. However, we have mentioned before that a large return per trade does not always means a large annual return. If the number of trading days when one is able to find these stocks is small then the annual return will be smaller. One needs to optimize criteria of stock selection to strike a balance between the numbers of stocks per year, which can be found for trading, the stock holding period, and the average return per trade. Examples of balanced trading strategies with low risk to return ratios can be found in our other publication "Short-term trading analysis" or the Text Level- 2 on the website http://www.stta-consulting.com.

We have now completed our description of the analytical methods, which can be used for improving trading strategies. We hope that our publication will help you to perform the analysis of your own trading strategy.

We have tried to describe very complicated questions as simply as possible. For this reasons many important points were but briefly described. We are always ready to help you to clarify our ideas. Feel free to ask us any questions. We would also very much appreciate it if you write us about other problems related to stock trading which you feel worth analyzing.

